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THE SMOOTHING AND THE DIGITAL PROCESSING OF LANGMUIR PROBE CHARACTERISTIC

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One of the most important parameter of a plasma source is the electron energy distribution function (EEDF), as it contains all the information about electron temperature, electron number density and resonant processes taking place inside the plasma. Here is discussed a method based on Druyvesteyn theory for computing the EEDF from Langmuir probe measurements. Also a method for choosing the optimum value of the parameter involved in Langmuir probe characteristic processing is presented.

Introduction

One of the most important parameter of plasmas is the electron energy distribution function (EEDF), as it contains all the information about electron temperature and electron number density. It also contains information about resonant processes taking place inside the plasma. As the EEDF can be linked to the external parameters of the plasma source (geometrical dimensions, gas pressure and flow, dissipated power etc.), we can eventually use those external parameters as guiding parameters in plasma processing of materials.

Druyvesteyn [1] showed in 1930 that the EEDF could easily be computed from the second derivative of the voltage-current characteristic of a Langmuir probe. Since then, numerous methods were employed in order to extract the EEDF from the voltage-current characteristic of a Langmuir probe, either by using analog devices or by using numerical methods. When numerical methods are involved, usually the EEDF is obtained after two step [1]: the first one gives the second derivative of the electronic current (which is proportional with the electron energy probability function - EEPF) and the second one gives the EEDF. This conducts to a very important problem in numerical processing of experimental data: if erroneously applied, all numerical methods will introduce distortions in the results and will point to false conclusions.

A widely used method for obtaining the second derivative of a function gift by a set of experimental data is the one proposed back in 1969 by Savitzky and Golay [2]. In this paper we analyze how the double derivative of an experimental dataset is changed as the only parameter involved in the computations changes its value.

Theory

The EEDF can be computed from the second derivative of the voltage-current characteristic of a Langmuir probe [1]:

$$EEDF(\varepsilon)\big|_{\varepsilon=eV_p} = \left(\frac{4}{A_p \cdot e^2}\right)\cdot\left(\frac{m_e \cdot V_p}{2e}\right)^{1/2} \cdot \frac{d^2 I_e}{dV_p{}^2} \qquad (1)$$

where $\mathbf{A_p}$ is the area of the probe, $\mathbf{V_p}$ is the probe potential and $\mathbf{I_e}$ is the electronic current driven by the probe from plasma. $\mathbf{I_e}$ is equal to the net current driven by the probe ($\mathbf{I_p}$) from which is subtracted the ionic saturation current ($\mathbf{I_i}$). Usually $\mathbf{I_i}$ is assumed to be a constant and so we can say that the EEDF is obtained not from the second derivative of $\mathbf{I_e}$ with respect to $\mathbf{V_p}$ but from the second derivative of $\mathbf{I_p}$ with respect to $\mathbf{V_p}$. Later we will see that we need to take into account $\mathbf{I_i}$ as it specifies the upper limit of the electron energy range for which EEDF can be computed using this method. It can be easily seen that the only problem in computing EEDF is the computation of the second derivative of $\mathbf{I_e}$ (or $\mathbf{I_p}$) with respect to $\mathbf{V_p}$.

Let us presume we have an experimental dataset comprised from **n** pairs of experimental points $\left\{\left(X^i, Y^i\right)\right\}$, $i = 1 \ldots n$. Let's also presume that this dataset contains, apart of the useful information, a certain amount of noise produced by the measurement electronics and by the statistical fluctuations of the recorded physical parameter. All the methods used for computing the second derivative have to take into account that in order to obtain valid results we need to eliminate this noise from the dataset. A simple assumption is that we deal with white noise.

A simple method for computing the second derivative is comprised from the following steps: (a) smooth the dataset; (b) compute the first derivative; (c) create a new dataset by smoothing the first derivative and (d) compute the first derivative of the newly created dataset. The smoothing of the dataset is made by averaging neighboring data using a Gaussian weighting. The first derivative is assumed to be equal to the first derivative of a spline function created from the dataset. This method for computing the second derivative has some disadvantages. First, the large number of processing steps makes difficult to track the influence of any particular operation. This will make difficult the interpretation of the results as it will be impossible to separate the real structure of the second derivative from possible artifacts. Second, if the noise recorded in the dataset is large and if the Gaussian weighting function is narrow, the second derivative will have sharp peaks alternating with deep gaps, thus rendering useless the results. Alternatively, if the Gaussian weighting function is broad, all the small features of the second derivative will be erased, leaving almost nothing to work on.

A much less destructive and one of the most widely used [3-5] method of computing the second derivative is the one devised by Savitzky and Golay [2]. Let' s take into account the same dataset as above: **n** pairs of experimental points $\left\{\left(X^i, Y^i\right)\right\}$, $i = 1 \ldots n$. In a given point of the dataset ($\left(X^j, Y^j\right)$) the second derivative is assumed to be equal to the second derivative of a function that fits a section of the dataset centered on the given point ($\left\{\left(X^{j-m}, Y^{j-m}\right), \ldots, \left(X^j, Y^j\right), \ldots, \left(X^{j+m}, Y^{j+m}\right)\right\}$, $m \in N$, $m \geq 1$, $m < \frac{n}{2}$). The number of experimental points that forms the section centered on the given point is **2m+1**. It can be seen that this method cannot give information about the second derivative for the points with indices $j < m$ and $j > n - m$. Remembering that our target is to compute the second derivative we can say that the simplest function that can be used to fit sections of the dataset is a second-degree polynomial. For a given point in the dataset ($\left(X^j, Y^j\right)$, $j > m$, $j < n - m$), the second derivative $\left.\frac{d^2X}{dY^2}\right|_j$ is obtained by fitting the section $\left\{\left(X^{j-m}, Y^{j-m}\right), \ldots, \left(X^j, Y^j\right), \ldots, \left(X^{j+m}, Y^{j+m}\right)\right\}$ with the function $f(x) = a \cdot x^2 + b \cdot x + c$. In this situation $\left.\frac{d^2X}{dY^2}\right|_j = 2a$. The value of the parameter **m** has to be established for each experiment as it depends on the number of experimental points in the dataset (**n**) and on the level of noise recorded along the data.

Experiment and discussion

We recorded Langmuir probe characteristics in the plasma of the positive column of a DC glow discharge. Ne at 1 torr was used as working gas knowing that the EEDF can be approximated well with a Maxwellian one. The discharge current of the DC glow discharge was 25 mA. A cylindrical probe having 3 mm in length and 0.5 mm in diameter was used. The probe voltage – current characteristic was recorded by varying the probe potential with respect to the anode potential in steps of 0.5 V, except for the ionic region where a 5 V step was used.

After processing the probe characteristic using the Langmuir theory we found the electron temperature and the electron number density to be those presented in the first column of table 1.

Table 1. Comparison between the values of n_e and T_e obtained from the same probe voltage – current characteristic using the classical Langmuir theory and the Druyvesteyn theory

	from classical Langmuir theory	**from Druyvesteyn theory, using the Savitzky – Golay smoothing & differentiating algorithm** m = 4	m = 5	m = 6
T_e (K)	**28900**	**29000**	**29000**	**29000**
n_e (m^{-3})	**$2.80*10^{15}$**	**$2.43*10^{15}$**	**$2.41*10^{15}$**	**$2.39*10^{15}$**

In order to compute the EEDF for the same probe voltage - current characteristic, we processed it with a program that uses the Savitzky - Golay algorithm described above. The program is written in BASIC and employs two routines for least-squares fitting. One routine is used for fitting the ionic current with a straight line and the other one is used for fitting sections of the electronic voltage - current characteristic with parabolic lines. For values of **m** in the range [1,...,6], the expression $\left(\frac{4}{A_p \cdot e^2}\right)\cdot\left(\frac{m_e \cdot V_p}{2e}\right)^{1/2}\cdot\frac{d^2 I_e}{dV_p^{\,2}}$ was computed. The results are presented in fig.1 – fig.6 (dots). In each of these figures the continuous line depicts the Maxwellian EEDF which corresponds to the values of $\mathbf{T_e}$ and $\mathbf{n_e}$ computed from classical Langmuir theory.

In fig.1 – fig.3 the reverted triangles near energy axis points to energy values at which the second derivative took negative values. Because of these negative values the dotted curves presented in fig.1 – fig.3 cannot be accounted as EEDFs.

The dotted curves presented in fig.4 – fig.6 are fitted very well by the Maxwellian EEDF having the parameters determined from classical Langmuir theory. As can be seen from table 1, the values of $\mathbf{T_e}$ computed from EEDFs are identical for all the three values of **m** but a decrease of $\mathbf{n_e}$ comes out as the value of **m** increases. This can be explained knowing that the EEDF will always have a peak in the vicinity of the origin of energy scale and that a greater value of **m** means a more accented smoothing. The peak in EEDF around the origin of energy scale comes from the large number of electrons which have little energy and because at zero eV the EEDF falls to zero. A more accented smoothing will flatten this peak which means that the value of $\mathbf{n_e}$ computed from EEDF will always be smaller than the value of $\mathbf{n_e}$ computed from the classical Langmuir probe theory. Because the difference comes out only in the region of low energies of EEDF where the shape of EEDF can be predicted it is easy to rectify it.

As can be seen here, there is an optimal value for **m**, for which the computed EEDF resembles very well the one computed from the values of $\mathbf{T_e}$ and $\mathbf{n_e}$ given by the classical Langmuir probe theory. This optimal value can be found only by checking successive values of **m** starting at **m** = 1. The minimum value of **m** for which EEDF can be found is the one for which the double derivative does not have any sharp peaks or negative values anymore. Starting from this point the EEDF flattens and both $\mathbf{n_e}$ and $\mathbf{T_e}$ changes. This points to the conclusion that this minimum value of **m** is also the optimal one.

As mentioned earlier, by processing the probe current ($\mathbf{I_p}$) or the electronic current ($\mathbf{I_e}$), we obtain same values for the second derivative and consequently the same results for EEDF. However it is usefull to take into account the ionic saturation current ($\mathbf{I_i}$) and consequently process $\mathbf{I_e}$ as the fall to zero of $\mathbf{I_e}$ gives a hint about the voltage range on which the second derivative can be linked with EEDF: when $\mathbf{I_e}$ approaches zero, although continuous, the second derivative can not be linked anymore with EEDF. This happens because the order of magnitude of $\mathbf{I_e}$ is the same with the order of magnitude of the noise superimposed on the dataset and also because negative values of the electronic current have no physical meaning. In our experiment, the energy range over which we can compute EEDF is $\varepsilon \in (0,12]\,\mathrm{eV}$. This range is merked with a vertical line in fig.4 – fig.6.

Conclusion

We presented here a method for establishing the optimal value for the fitting parameter required by the Savitzky – Golay algorithm in order to compute EEDF from single probe measurements. The method was verified by comparing the values of $\mathbf{n_e}$ and $\mathbf{T_e}$ found using the Druyvesteyn theory with the values of $\mathbf{n_e}$ and $\mathbf{T_e}$ found using the Langmuir theory. The values were found to be in very good agreement.

References

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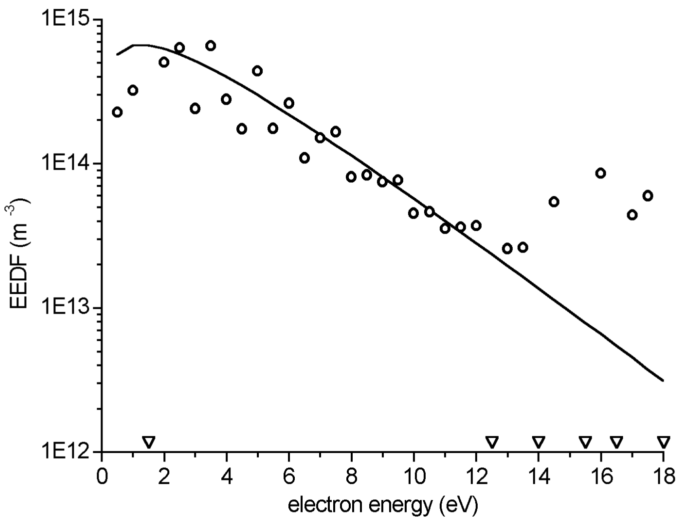
Fig. 1. The values of the expresion $\left(\frac{4}{A_p \cdot e^2}\right)\cdot\left(\frac{m_e \cdot V_p}{2e}\right)^{1/2} \cdot \frac{d^2 I_e}{dV_p{}^2}$ computed for **m = 1** (hollow dots) and a Maxwellian EEDF having the parameters ($\mathbf{n_e}$ and $\mathbf{T_e}$) computed from Langmuir probe theory (continuous line).



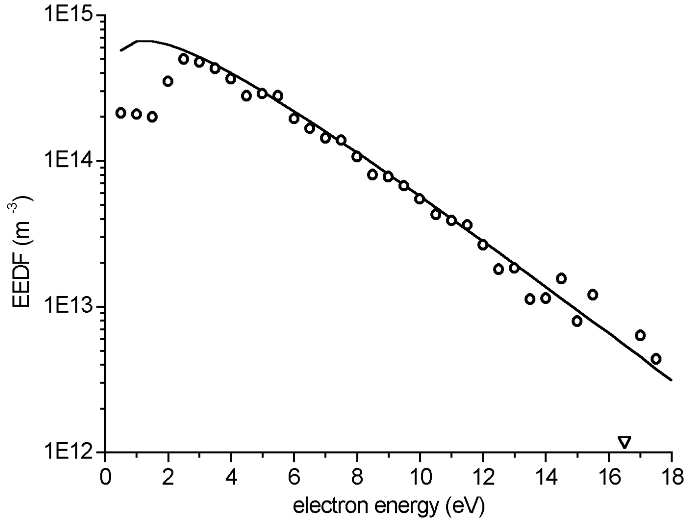


Fig. 2. The values of the expresion $\left(\frac{4}{A_p \cdot e^2}\right) \cdot \left(\frac{m_e \cdot V_p}{2e}\right)^{1/2} \cdot \frac{d^2 I_e}{dV_p^{\ 2}}$ computed for **m = 2** (hollow dots) and a Maxwellian EEDF having the parameters (**n_e** and **T_e**) computed from Langmuir probe theory (continuous line).

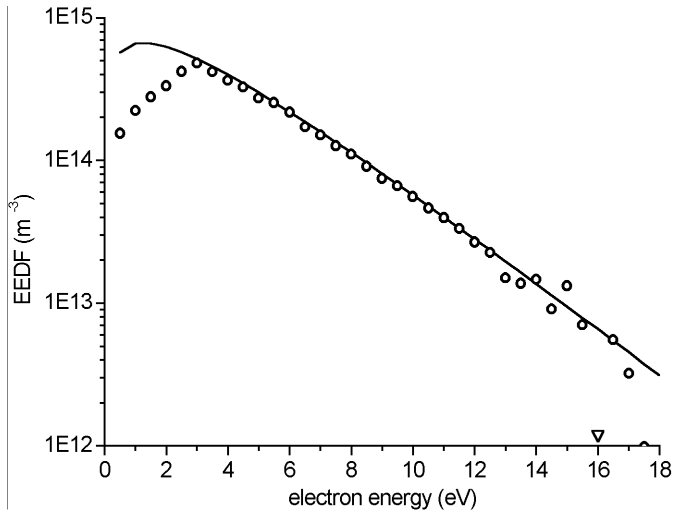


Fig. 3. The values of the expresion $\left(\frac{4}{A_p \cdot e^2}\right) \cdot \left(\frac{m_e \cdot V_p}{2e}\right)^{1/2} \cdot \frac{d^2 I_e}{dV_p^{\ 2}}$ computed for **m = 3** (hollow dots) and a Maxwellian EEDF having the parameters (**n_e** and **T_e**) computed from Langmuir probe theory (continuous line).

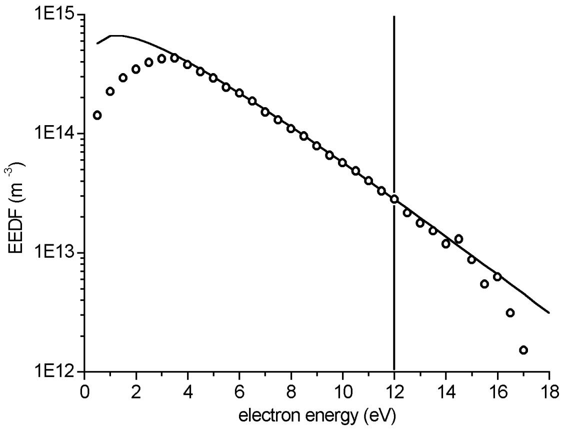


Fig. 4. The values of the EEDF (equation (1)) computed for **m = 4** (hollow dots) and a Maxwellian EEDF having the parameters (**n_e** and **T_e**) computed from Langmuir probe theory (continuous line).

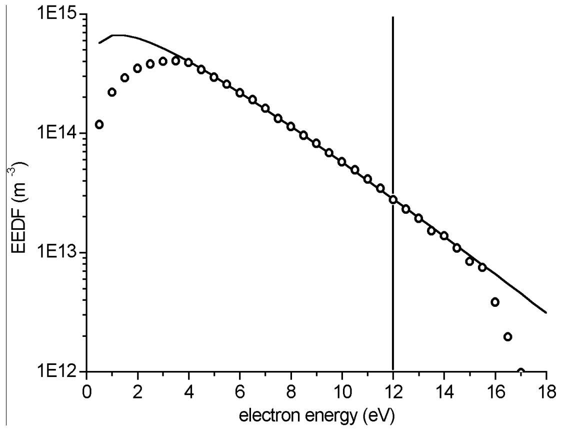


Fig. 5. The values of the EEDF (equation (1)) computed for **m = 1** (hollow dots) and a Maxwellian EEDF having the parameters (**n_e** and **T_e**) computed from Langmuir probe theory (continuous line).

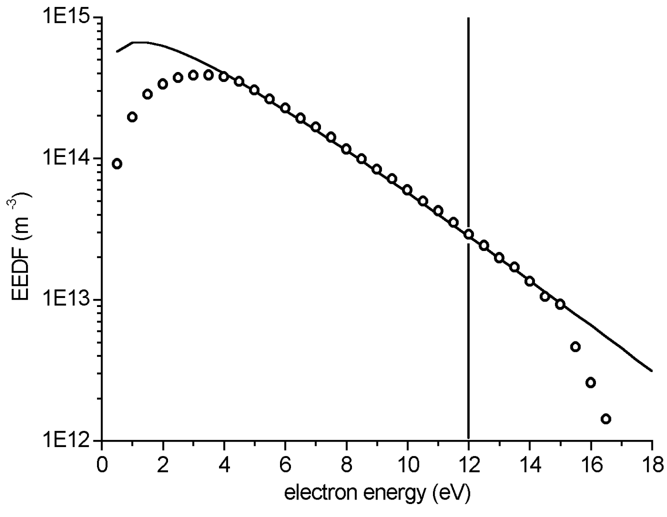


Fig. 6. The values of the EEDF (equation (1)) computed for **m = 1** (hollow dots) and a Maxwellian EEDF having the parameters (**n_e** and **T_e**) computed from Langmuir probe theory (continuous line).