Romanian Reports in Physics, Volume 55, Number 4, P. 549-554, 2003

True ternary fission

D. N. Poenaru[1,2], R. A. Gherghescu[1,2],
W. Greiner[2], Y. Nagame[3], J. H. Hamilton[4],
and A. V. Ramayya[4]
[1]Horia Hulubei National Institute of Physics and Nuclear Engineering,
P. O. Box MG-6, RO-077125 Bucharest, Romania
[2]Institut für Theoretische Physik der J. W. Goethe Universität, Pf 111932,
D-60054 Frankfurt am Main, Germany
[3]Advanced Science Research Center, Japan Atomic Energy Research Institute,
Tokai, Ibaraki 319-1195, Japan
[4]Department of Physics, Vanderbilt University, Nashville, TN 37235, USA

Abstract

Despite early theoretical predictions, the true ternary fission (with three identical fragments) was not experimentally detected until now. A method to study ternary fission, which allows to obtain the equilibrium shape without being necessary to specify a certain parametrization is presented. As an example, the method is applied to the ternary fission of ^{170}Yb.

1 Introduction

The complexity of different fission phenomena [1] still continues to be revealed. Particle accompanied fission was discovered in 1946 (see the reviews [2, 3]), but only recently, by using new methods of fission fragment identification [4], the "cold" processes α and ^{10}Be accompanied cold fission as well as the double and triple fine structure in a binary and ternary fission have been reported [5].

The importance of scission configuration for ternary fission [6] was repeatedly stressed in the past. In binary fission, the scission configuration is now better understood due to a longstanding effort of systematic analysis [7–14]. One knows three types of shape elongations: LDM governed mass symmetric elongated shape (deformation $\beta \simeq 1.65$); mass asymmetric deformation ($\beta \simeq 1.53$), and shell-influenced mass symmetric deformation ($\beta \simeq 1.43$).

The unified approach of cold binary fission, cluster radioactivity, and α-decay [1] was extended to cold ternary fission [15] and to multicluster fission [16]. One should recall that fission phenomena [17] were previously studied in the Institute of Atomic Physics, e.g. fission-isomers [18, 19], potential energy surfaces [20–23], and α-decay was interpreted as a cold fission phenomenon [24–27].

Theoretically it was pointed out by Present [28] in 1941 that Uranium tripartition would release about 20 MeV more energy than the binary one. In the present paper we consider the nuclear fragmentation into three identical or nearly identical fragments. In spite of having quite large Q values [29], this "true ternary fission" is a rather weak process; the strongest phenomenon remains α-particle-accompanied fission [3]. Experiments on so-called "symmetrical tripartition" were performed *e.g.* using the induced fission of ^{235}U by thermal neutrons [30], the induced fission of ^{238}U by intermediate-energy hellium ions [31], or spontaneous fission of ^{252}Cf [32]. An yield of 6.7 ± 3.0 per 10^6 binary fissions was reported by Rosen and Hudson [30] who employed a triple gas-filled ionization chamber and a suitable electronics including a triple coincidence circuit. Other "optimistic" results are mentioned by Iyer and Cobble [31], who tried radiochemical methods of identification at intermediate energy of excitation. While at high energy [33] implying bombarding heavy ions of several hundred MeV, a positive result may be accepted, it is not certain whether it comes out from a compund nucleus. The general conclusion [32] after measuring triple coincidences with detectors placed at 120° is rather pessimistic: except for excitation energies over 24 MeV, the true ternary yield is extremely low: under 10^{-8} per binary fission act.

2 Fission into identical fragments

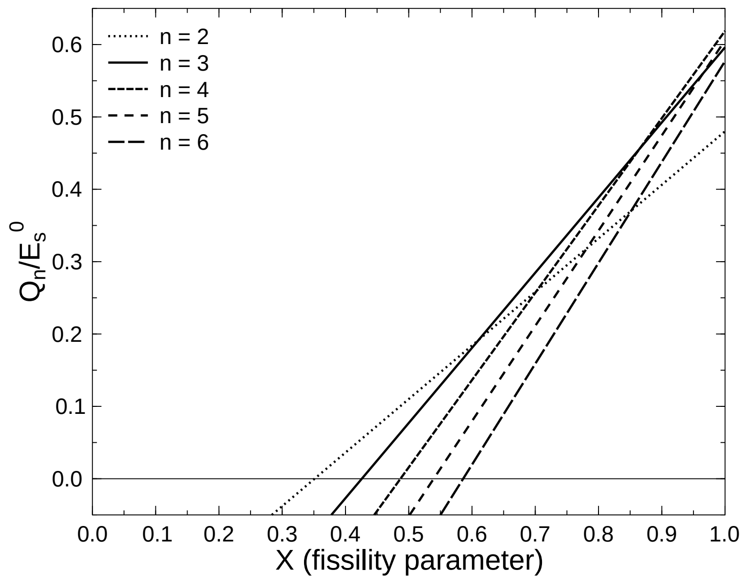

Figure 1: Q values divided by the surface energy of the parent nucleus versus fissility parameter, for fission into n identical spherical fragments as estimated within the liquid drop model.

In 1958 it was theoretically shown [34] on the basis of the liquid drop model [35] that for increasingly heavier nuclei, fission into three, then four and even five fragments becomes energetically more favourable than binary fission (see Fig. 1). One can take, as an approximation of the Q value, the energy difference between the sum of Coulomb and surface energies for the parent (superscript ^{0}) and n identical fission fragments (superscript ^{i})

$$Q_n \simeq (E_C^0 + E_s^0) - \sum_{i=1}^{n}(E_C^i + E_s^i) \tag{1}$$

where $n = 2$ for binary fission, $n = 3$ for ternary fission, etc. A linear dependence of Q_n on the (binary) fissility parameter, $X = E_C^0/(2E_s^0)$, of the form

$$Q_n/E_s^0 \simeq 1 - n^{1/3} + 2X(1 - n^{-2/3}) \tag{2}$$

has been obtained [34]. In Fig. 1 one sees that as the fissility parameter increases, fission into more than two equal fragments becomes energetically favored. At $X \geq 0.426$ tripartition becomes exothermic and for $X \geq 0.611$ the Q-value for fission into three identical fragments is larger than that for binary fission. The general trend, and sometimes even the absolute values of Q_2 and Q_3, are well reproduced by the above equation [29].

3 Equilibrium shapes

Recently, a method was developed [36], allowing to obtain very general shapes as a solution of an integro-differential equation without a need to give *apriori* any shape parametrization. This equation was derived by minimizing the potential energy with constraints (constant volume and given deformation parameter). The method allows to obtain straightforwardly the axially-symmetric surface shape for which the liquid drop energy, $E_{LDM} = E_s + E_C$, is minimum. By adding the shell corrections δE to the LDM deformation energy, $E_{def} = E_{LDM} + \delta E$, one can get minima at a finite value of the mass-asymmetry parameter for binary fission.

The nuclear surface equation with axial symmetry around z axis, expressed as $\rho = \rho(z)$ in cylindrical coordinates, should minimize the potential energy of deformation with two constraints: volume conservation, and given deformation parameter, α, assumed to be an adiabatic variable. The dependence on the neutron and proton numbers is contained in the surface energy E_s^0, the fissility parameter $X = E_C^0/(2E_s^0) = [3Z^2e^2/(5R_0)]/2[a_s(1-\kappa I^2)A^{2/3}]$, as well as in the shell correction of the spherical nucleus δE^0. The radius of spherical nucleus is $R_0 = r_0 A^{1/3}$ with $r_0 = 1.2249$ fm, and $e^2 = 1.44$ MeV·fm is the square of electron charge.

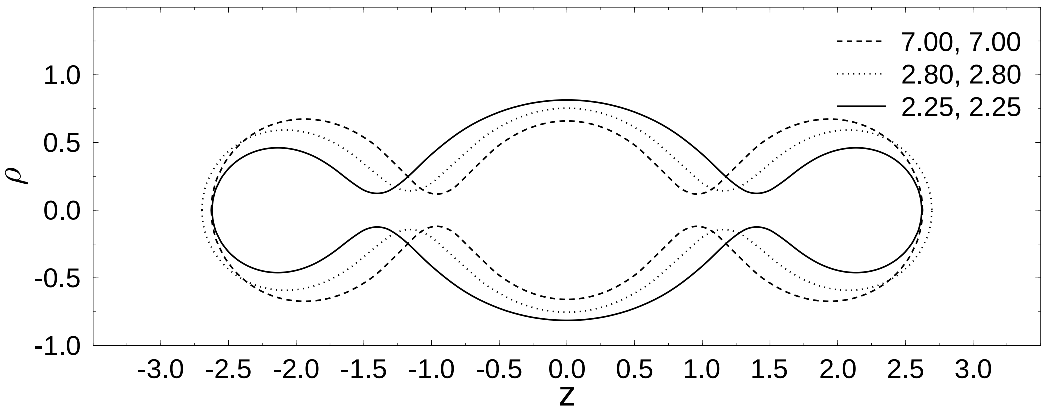

Figure 2: Nuclear shapes obtained by solving an integro-differential equation [36] for $n_L = n_R = 4$ and $d_L = d_R = 2.30; 2.70$ and 4.00. The binary fissility $X = 0.60$ corresponds to ^{170}Yb.

The corresponding variational problem can be reduced to the following equation

$$\rho\rho'' - \rho'^2 - [\lambda_1 + \lambda_2|z| + 10XV_s(z,\rho)]\,\rho(1+\rho'^2)^{3/2} - 1 = 0 \tag{3}$$

where $\rho' = d\rho/dz$, $\rho'' = d^2\rho/dz^2$, and V_s is the Coulomb potential on the nuclear surface. This is an integro-differential equation because the Coulomb potential is expressed as an integral [22, 23]. We used the following relationships for the principal radii of curvature $\mathcal{R}_1 = \tau\rho$, ${\mathcal{R}_2}^{-1} = -\rho''/\tau^3$, in which $\tau^2 = 1 + \rho'^2$. In this eq. λ_1 and λ_2 are Lagrange multipliers corresponding to the constraints of volume conservation (or given mass asymmetry if the voulume is conserved in each "half" of the nucleus) and determined value of deformation parameter α, defined as the distance between centers of mass of the fragments lying at the left hand side and right hand side of the plane $z = 0$, respectively: $\alpha = |z_L^c| + |z_R^c|$. This definition allows to reach all intermediate stages of deformation from one parent nucleus to two fragments by a continuos variation of its value. The position of separation plane, $z = 0$, is given by the condition: $(d\rho/dz)_{z=0} = 0$. Lengths are given in units of R_0, Coulomb potential in units of Ze/R_0, and energy in units of the surface energy E_s^0. One can calculate for every value of α the deformation energy $E_{def}(\alpha)$. The particular value α_s for which $dE_{def}(\alpha_s)/d\alpha = 0$ corresponds to the extremum, i.e. the shape function describes the saddle point (or the ground state), and the unconditional extremum of the energy is the fission barrier. The other surfaces (for $\alpha \neq \alpha_s$) are extrema only with condition $\alpha =$ constant. In this way one can compute the deformation energy function of two variables: elongation and mass asymmetry. Details are given elsewhere [36], including the definition of input parameters $d_L = d_R$ and $n_L = n_R$ for relection-symmetric shapes. Here L and R label the left and right parts.

The elongated shapes for ternary fissions are shown in Figure 2. For shapes with three fragments and two necks ($n_L = n_R = 3$) from 2.25 to 2.80 and 7.00 the deformation α increases from 1.650 to 2.306 and 2.730. In the same time the elongation is initially increased from 5.234 to 5.392 and then decreased to 5.24; the fragment radii are 0.461/0.814/0.461, 0.592/0.753/0.592, and 0.673/0.659/0.673, leading to decreasing energies in units of E_s^0 from 0.165 to 0.150 and 0.134. The last configuration with $E/E_s^0 = 0.134$ is not far from a "true ternary-fission" in which the three fragments are almost identical: ${}^{170}_{70}\mathrm{Yb} \rightarrow {}^{56}_{23}\mathrm{V} + {}^{56}_{23}\mathrm{V} + {}^{58}_{24}\mathrm{Cr}$ and the Q-value is 83.639 MeV. One may compare the above E/E_s^0 value with the touching-point energy of these spherical fragments $(E_t - Q)/E_s^0 = 0.239$. It is larger, as expected, because of the finite neck of the shapes in Figure 2. For α-accompanied fission of ${}^{170}\mathrm{Yb}$ with two ${}^{83}_{34}$fragments $Q = 87.484$ MeV is larger and the touching point energy $(E_t - Q)/E_s^0 = 0.103$ is lower. A lower $Q = 70.859$ MeV and higher energy barrier $(E_t - Q)/E_s^0 = 0.147$ is obtained for ${}^{10}\mathrm{Be}$ accompanied fission of ${}^{170}\mathrm{Yb}$ with ${}^{80}_{33}\mathrm{As}$ fission fragments.

We should stress again that if one is interested to estimate the yield in various fission processes, one has to compare the potential barriers and not the Q-values. Our results are in agreement with preceding calculations [37] showing also preference for prolate over oblate shapes.

The shapes plotted in Figure 2 are produced for various values of input parameters; only one of these shapes corresponds to the saddle-point. One should not be confused about the unexpected shape with $d_L = d_R = 2.25$ having a large fragment between two smaller ones; it is the result of the low input value of $d_L = d_R$.

By performing dynamical calculations, Hill arrives in his thesis and in [38] at elongated shapes with pronounced necks looking more encouraging. In conclusion the "true" ternary sponataneous fission is an extremely rare phenomenon which could be only observed in the future with a very much improved experimental sensitivity.

Acknowledgement

This work was partly supported by Japanese Society for the Promotion of Science (JSPS), Tokyo, by the Japan Atomic Energy Research Institute (JAERI), Tokai, by UNESCO (UVE-ROSTE Contract 875.737.2), by the Center of Excellence IDRANAP under contract ICA1-CT-2000-70023 with European Commission, Brussels, and by the Ministry of Education and Research, Bucharest. One of us (DNP) would like to acknowledge the hospitality and financial support received during his research stage in Tokai, when the present work was completed.

References

[1] D. N. Poenaru, M. Ivaşcu, and W. Greiner, in *Particle Emission from Nuclei, Vol. III: Fission and Beta-Delayed Decay Modes*, (CRC, Boca Raton, 1989), Chap. 7, pp. 203–235.

[2] C. Wagemans, Chap. 3, in [1], pp. 63–97.

[3] M. Mutterer and J. P. Theobald, in *Nuclear Decay Modes*, (Institute of Physics, Bristol, 1996), Chap. 12, pp. 487–522.

[4] J. H. Hamilton *et al.*, Prog. Part. Nucl. Phys. **35**, 635 (1995).

[5] A. V. Ramayya *et al.*, Phys. Rev. Lett. **81**, 947 (1998); Phys. Rev. C *57*, 2370 (1998).

[6] A. Gavron, Phys. Rev. C **11**, 580 (1975).

[7] Y. Nagame *et al.*, Phys. Lett. B **387**, 26 (1996).

[8] Y. Nagame *et al.*, Radiochimica Acta **78**, 3 (1997).

[9] I. Nishinaka *et al.*, Phys. Rev., C **56**, 891 (1997).

[10] T. Ohtsuki, Y. Nagame, and H. Nakahara, in *Heavy Elements and Related New Phenomena*, (World Sci., Singapore, 1999), Vol. I, Chap. 13, pp. 507–535.

[11] Y. Nagame *et al.*, J. Radioanal. Nucl. Chem. **239**, 97 (1999).

[12] Y. L. Zhao *et al.*, Phys. Rev. Lett. **82**, 3408 (1999).

[13] Y. L. Zhao, Y. Nagame, I. Nishinaka, K. Sueki and H. Nakahara, Phys. Rev., C **62**, 014612(9) (2000).

[14] Y. Nagame *et al.*, Radiochimica Acta **89**, in print (2001).

[15] D. N. Poenaru, B. Dobrescu, W. Greiner, J. H. Hamilton, and A. V. Ramayya, J. Phys. G **26**, L97 (2000).

[16] D. N. Poenaru, W. Greiner, J. H. Hamilton, A. V. Ramayya, E. Hourany and R. A. Gherghescu, Phys. Rev., C **59**, 3457 (1999).

[17] M. Ivaşcu and D. N. Poenaru, *Deformation Energy and Nuclear Shape Isomers (in Romanian)* (Editura Academiei, Bucharest, 1981).

[18] G. N. Flerov *et al.*, Nucl. Phys. **97**, 444 (1967).

[19] D. N. Poenaru, Ann. Phys. (Paris) **2**, 133 (1977).

[20] D. N. Poenaru, Report CRD-59, Institute of Atomic Physics, Bucharest (unpublished) (1975).

[21] D. N. Poenaru, M. Ivaşcu, and D. Mazilu, J. Phys. G **5**, 1093 (1979).

[22] D. N. Poenaru and M. Ivaşcu, Comp. Phys. Communic. **16**, 85 (1978).

[23] D. N. Poenaru, M. Ivaşcu, and D. Mazilu, Comp. Phys. Communic. **19**, 205 (1980).

[24] D. N. Poenaru, M. Ivaşcu, and A. Săndulescu, J. Phys. G **5**, L169 (1979).

[25] D. N. Poenaru, M. Ivaşcu, and D. Mazilu, J. Phys. Lett. **41**, L589 (1980).

[26] D. N. Poenaru, M. Ivaşcu, and D. Mazilu, Comp. Phys. Communic. **25**, 297 (1982).

[27] I. V. Poplavsky, Izv. Akad. Nauk, Ser. Fiz. **61**, 769 (1997); **64**, 422 (2000).

[28] R. D. Present, Phys. Rev. **59**, 466 (1941).

[29] D. N. Poenaru, W. Greiner, and R. A. Gherghescu, Atomic Data Nucl. Data Tab. **68**, 91 (1998).

[30] L. Rosen and A. M. Hudson, Phys. Rev. **78**, 533 (1950).

[31] R. H. Iyer and J. W. Cobble, Phys. Rev. **172**, 1186 (1968).

[32] P. Schall, P. Heeg, M. Mutterer, and J. P. Theobald, Phys. Lett., B **191**, 339 (1987).

[33] R. L. Fleischer, P. B. Price, R. M. Walker, and E. L. Hubbard, Phys. Rev. **143**, 943 (1966).

[34] W. J. Swiatecki, in Proc. 2nd U. N. Int. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1-13 Sept, 1958, (United Nations, Geneva, 1958) p. 248-272.

[35] W. D. Myers and W. J. Swiatecki, Nucl. Phys. A **81**, 1 (1966).

[36] D. N. Poenaru, W. Greiner, Y. Nagame, and R. A. Gherghescu, in *Advances in Heavy Element Research* (Proc. of the 2nd International Symposium on Advanced Science Research, Tokai, Japan, 2001), Ed. by Y. Nagame *et al.*, J. Nucl. Radiochem. Sci., Japan. in print (2002).

[37] H. Diehl and W. Greiner, Nucl. Phys., A **229**, 29 (1974).

[38] D. L. Hill, in Proc. 2nd U. N. Int. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1-13 Sept, 1958, (United Nations, Geneva, 1958) p. 244-247.