K-shell photoionization in the presence of a laser field

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Abstract

We study the atomic photoeffect due to a monochromatic radiation of low intensity, in the presence of another intense monochromatic electromagnetic field (the laser). Using a perturbed wavefunction, including second-order corrections in the intense field amplitude, we derive the general equations which describe one-photon absorption from the first source, simultaneously with a net exchange of N photons ($N = 0, \pm 1 \pm 2$) with the laser. In the case of a hydrogenlike atom, we derive analytic expressions for the transition amplitudes, valid in the lowest order in the Coulomb field strength. The numerical results are given in the case of a high frequency laser.

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1 Introduction

Atomic physics in the presence of a laser field is a domain recording constant progress in the last twenty years. Unexpected phenomena have been discovered, as above threshold ionization, high-order harmonic generation and multiple ionization. And new tools of investigation have resulted; for instance, recent achievements in the high harmonic generation made possible the development of coherent radiation sources in the X-ray domain. With such a source it was possible to detect in 1996 the laser-assisted single ionization of helium in the case of phase-coherent fields [1, 2]. In the experiment performed at Palaiseau [1], the soft-X-ray pulses were generated as high-order harmonics of a Ti:sapphire laser, which provided also the infrared (IR) fundamental frequency of 800 nm. The ionization was due to the 21st harmonic (38 nm). The IR laser modifies the photoelectron spectrum obtained in its absence; the appearance of sidebands due to absorption and emission of laser photons during X-ray ionization was recorded.

New intense radiation beams in the VUV- and XUV-wavelength regime generated by free-electron laser are expected to open a new area of work for both experiment and theory. The possibility of inner-shell ionization, generating "hollow ions" was recently envisaged [3].

Theoretical work in the laser-atom interaction area is marked by the fact that perturbation theory, the traditional method in the study of atom-electromagnetic field interaction, is not able to describe the majority of experimental facts. So, this method was replaced by other approaches, as Floquet theory or the direct integration of time dependent Schrödinger equation (TDSE), or it was modified in a manner convenient for fields of moderate intensities. The influence of the field is essential in the case of electrons in the continuum states.

Our paper is devoted to one-photon absorption from a source of low intensity in the presence of a second monochromatic source of moderate intensity, much less the atomic unit of intensity.

X-ray absorption in atoms in the presence of a laser field was considered in several theoretical studies starting with a calculation of Freund in 1973 of the two-photon absorption coefficient, one photon from the X-ray source, the other from the laser. The calculation [4] was done in perturbation theory and has used a plane wave for the final state of the electron and a hydrogenic wave function for the initial state. But, the situation that needed investigation was that of an intense laser field, which cannot be treated in perturbation theory. This is why Tzoar and Jain [5] have replaced the plane wave by a Volkov solution; Coulomb effects due to the nucleus on the final electron state and dressing effects on the initial electron state were still neglected. Further, in a study devoted to Compton scattering in the presence of coherent electromagnetic radiation[6], Jain and Tzoar have introduced the Coulomb-Volkov approximation, which partially takes in to account Coulomb effects. This approximation was used by Ferrante *et al.* [7] who found that multiphoton emissions prevail over the absorptions for very soft X-rays (150 eV) and 0.117 eV laser photons.

An improvement in the description of the final electron in the atomic and the laser fields was proposed by Bannerji and Mittlemann [8] and has been developed by Joachain *et al.* [9] in a study of $(e, 2e)$ collisions in the presence of a laser field. Later, this last approach was adopted by Cionga *et al.* [10] in a calculation of the satellites of first order, a calculation which also includes the first-order laser effect on the initial state. The whole procedure, which can be described as semi-perturbative, is adequate for not too intense lasers. The approach was used recently, in a slightly modified version, by Milosevic and Ehlotzky [11] in a study of photoionization in the presence of a bichromatic laser field. An account of other calculations on photoionization in the presence of a laser field can be found in a recent report by Ehlotzky [12].

In our calculation, we adopt the formalism of Joachain *et al.* [9] and we make a step farther, including second-order effects of the laser on both the initial (bound) state and final (continuum). Recently, the same method was used in a calculation of laser assisted bremsstrahlung [14], implying these effects on intial and final continuum states. To our knowledge these second-order effects were considered up to know only by Shakeshaft and coworkers [13] which, after deriving adequate equations for a non-perturbative approach, treat, in perturbation theory, the special case in which there is only one type of photons, the one-photon ionization rate being modified by the simultaneous exchange of a pair of photons (absorption and emission).

The paper is organized as follows. In Sect. 2, we present the general equations we use, based on the assumption that the ionization is due to the absorption of a X-ray photon from a source of low intensity, and that another monochromatic source of lower frequency and moderate intensity electromagnetic radiation is present. We give the expressions for the transition amplitudes corresponding to the X-ray absorption simultaneous with an exchange of N laser photons, for $N = 0, \pm 1, \pm 2$.

In the following sections we refer to atomic hydrogen, a restriction which allows further analytic manipulations. The main mathematical object to be evaluated is a third order tensor corresponding to three-photon transitions. We adopt zeroth-order Born approximation for its evaluation, as described in Sect. 3. The complicated part of the analytic calculation, described in Sect. 4, comes from the stimulated radiative correction to the ground state.

In the numerical calculations, presented in Sect.5, we have taken $\hbar\omega_X > \hbar\omega \gg \mid E_1 \mid$ (X-ray range), with E_1 the ground state energy. These restrictions are connected with the Born approximation we use. So, our results apply for high-frequency lasers, which will be available in the future [3], and not for the usual ones in the infrared or optical ranges. Because all the calculation is nonrelativistic, we have also $\hbar\omega_X \ll m_e c^2$, with m_e-electron mass, c-the velocity of light.

The calculation here, using the zeroth order Born approximation, is a first step to a more elaborate study including Coulomb effects.

2 Assisted X-ray absorption amplitudes

The system we consider in this section is an electron (charge $e < 0$) in an atomic field $V(\mathbf{r})$, interacting with two monochromatic sources of electromagnetic radiation. In the velocity gauge the Hamiltonian is

$$H = H_a - \frac{e}{m_e}\mathbf{A}\cdot\mathbf{P} + \frac{e^2}{2\,m_e}\mathbf{A}^2 \tag{1}$$

where H_a is the Hamiltonian of the electron in the absence of the electromagnetic fields, $\mathbf{P}$ is the momentum operator and the potential vector $\mathbf{A}$ is the sum,

$$\mathbf{A} = \mathbf{A}_X + \mathbf{A}_L \tag{2}$$

of a term describing the X-ray field and a term describing the laser field. We adopt the dipole approximation for both fields, in which case the potential vector depends on time only. This allows the elimination of the $\mathbf{A}^2$ term from the Hamiltonian through the usual procedure of extracting the time-dependent phase factor $F(t) = \exp[-(ie^2/2m_e)\int_{t_0}^{t}\mathbf{A}^2(t')\,dt']$. This factor will not be present in the transition rate, so it will not be written in the transition amplitude. Then, the evolution of a system initially ($t = t_0$) in the ground state is contained in the wavefunction $\mid \Psi(t) >= U(t,t_0) \mid E_1 >$, with $\mid E_1 >$ the ground state energy eigenvector, where the time-evolution operator U is associated to the Hamiltonian (1) without the $\mathbf{A}^2$ term . The wavefunction generates transition amplitudes A_f to different final states $\mid f >$, according to $A_f =< f \mid \Psi(t) >$. For a weak X-ray field, one can use the approximation

$$U(t,t_0) \approx U_{\mathrm{a}L}(t,t_0) + \frac{1}{i\hbar}\int_{t_0}^{t} U_{\mathrm{a}L}(t,t')\,H_X U_{\mathrm{a}L}(t',t_0)\,dt' \tag{3}$$

$H_X = -(e/m_e)\,\mathbf{A}_X\cdot\mathbf{P}$, and $U_{\mathrm{a}L}(t,t_0)$ is the time evolution operator associated to the electron in the atomic potential and the laser field only, i.e., to the Hamiltonian

$$H_{\mathrm{a}L} = H_a - \frac{e}{m_e}\mathbf{A}_L\cdot\mathbf{P}\,. \tag{4}$$

The first term in the expression of U gives transitions we are not interested in here, in which no X-ray photon is absorbed. As a consequence, we consider the term

$$A_f^X = \frac{1}{i\hbar}\int_{t_0}^{t} < f \mid U_{\mathrm{a}L}(t,t')\,H_X\,U_{\mathrm{a}L}(t',t_0) \mid E_1 > \,dt'. \tag{5}$$

As noticed previously [8], the vectors $U_{\mathrm{a}L}(t',t_0) \mid E_1 >$ and $U_{\mathrm{a}L}(t',t) \mid f >$ satisfy in their dependence of t' the TDSE for the Hamiltonian (4). At the right stage, the limits $t_0 \to -\infty$ and $t \to \infty$ are taken. So, we have

$$A_f^X = \frac{1}{i\hbar}\int_{t_0}^{t} < \Psi_f(t') \mid H_X \mid \Psi_1(t') > \,dt', \tag{6}$$

with the perturbed vectors $\mid \Psi_1 >$ and $\mid \Psi_f >$ defined by

$$\mid \Psi_1(t) >= U_{\mathrm{a}L}(t,-\infty) \mid E_1 >\,, \qquad \mid \Psi_f(t) >= U_{\mathrm{a}L}(t,\infty) \mid f >\ . \tag{7}$$

We use a second-order approximation [15] for the ground state evolution in a monochromatic electromagnetic field (called the laser here), namely

$$\mid \Psi_1(t')\rangle = \mathcal{N}_1\,[\,\mid E_1\rangle + \sum_{\nu=-2}^{2} e^{-i\nu\omega t'}\,|g_\nu\rangle\,]\,e^{-\frac{i}{\hbar}(E_1+\Delta E_1)t'}\,, \tag{8}$$

where ΔE_1 is the dynamic Stark-shift. To the normalisation constant $\mathcal{N}_1$, not considered in [15], we devote Appendix A. In the case of a linearly polarized laser field described by [16]

$$\mathbf{A}_L(t) = A_0\, cos(\omega t)\, \mathbf{s}\,, \tag{9}$$

where $\mathbf{s}$ is the polarization vector (real and of unity length), with the notation

$$\alpha_0 = -\frac{e}{m_e\,\omega}\, A_0\, \mathbf{s} = \alpha_0\, \mathbf{s}\,, \tag{10}$$

the expressions of the ket-vectors in (8) are

$$|g_{\pm 1}\rangle = \frac{\omega}{2}\, \alpha_0\, G_a(E_1 \pm \hbar\omega)\, \mathbf{s}\cdot\mathbf{P} \mid E_1\rangle\;, \tag{11}$$

$$|g_{\pm 2}\rangle = \frac{\omega^2}{4}\, \alpha_0^2\, G_a(E_1 \pm 2\hbar\omega)\, \mathbf{s}\cdot\mathbf{P}\, G_a(E_1 \pm \hbar\omega)\, \mathbf{s}\cdot\mathbf{P}\; \mid E_1\rangle\;, \tag{12}$$

$$|g_0\rangle = \frac{\omega^2}{4}\alpha_0^2 G_a^{red}(E_1)\, \mathbf{s}\cdot\mathbf{P}\, [G_a(E_1+\hbar\omega) + G_a(E_1-\hbar\omega)]\mathbf{s}\cdot\mathbf{P} \mid E_1\rangle\,, \tag{13}$$

$$\Delta E_1 = \frac{\omega}{2}\; < E_1 \mid \mathbf{s}\cdot\mathbf{P} \mid g_1 + g_{-1} >\;, \tag{14}$$

where G_a and G_a^{red} denote, respectively, the retarded Green operator $G_a^{(+)}$ and the reduced Green operator associated to the atomic Hamiltonian H_a.

For the final state vector $\mid f >$ in $\mid \Psi_f >$, we take a continuum incoming eigenstate, $\mid E\,\mathbf{n}\,-\rangle$, normalized in the energy and solid angle scales. The corresponding asymptotic momentum is denoted by $\mathbf{p} = p\,\mathbf{n}$. Because the effect of the laser field is important on the continuum state, we adopt the approximation that factors out the Volkov factor

$$\xi_V(\mathbf{p},t) = \exp[-\frac{i}{\hbar}\, \alpha_0\, \mathbf{s}\cdot\mathbf{p}\, sin(\omega\, t)], \tag{15}$$

and includes corrections up to second order in the laser amplitude,

$$\mid \Psi_f(t')\rangle = \mathcal{N}_f\, \xi_V(\mathbf{p},t')\, [|\, E\,\mathbf{n}\,-\rangle + \sum_{\nu=-2}^{2} e^{-i\,\nu\,\omega t'}\, |\tilde{f}_\nu\rangle\,]\, e^{-\frac{i}{\hbar}Et'}\;, \tag{16}$$

The normalisation constant $\mathcal{N}_f$ ensures for the perturbed vector the normalization in the energy and solid angle scales. The vectors $|\tilde{f}_\nu\rangle$ can be formally obtained from $\mid g_\nu\rangle$, by replacing E_1 by $E > 0$, the energy in the continuum, $\mid E_1\rangle$ by $\mid E\,\mathbf{n}-\rangle$, and $\mathbf{s}\cdot\mathbf{P}$ by $\mathbf{s}\cdot(\mathbf{P}-\mathbf{p})$. Also, both the Green operator and the reduced Green operator should be meant now as the retarded Green operator.

We come back to (5) and replace $\mathbf{A}_X$ by

$$\mathbf{A}_X = A_{0X}\, [\, \mathbf{s}_X\, \exp(-i\omega_X t) + \mathbf{s}_X^*\, \exp(i\omega_X t)\,]\,, \tag{17}$$

with $\mathbf{s}_X$ the polarization vector of the X-radiation. The equations below allow any possibility for the polarization, i.e., $\mathbf{s}_X$ might be complex, with $\mathbf{s}_X^*\cdot\mathbf{s}_X = 1$. Then, we replace the vectors with the expressions described before. The Volkov factor is Fourier expanded as

$$\xi_V(\mathbf{p},t) = \sum_{m=-\infty}^{\infty} e^{-im\omega t}\, J_m(z)\,, \qquad z = \alpha_0\cdot\mathbf{p}/\hbar \tag{18}$$

with J_m the Bessel function of integer order.

Finally, when $t_0 \to -\infty$ and $t \to \infty$, the integrals in (5) lead to $\delta-$functions, so the transition amplitude is a formal series of $\delta-$functions,

$$A_f^X = 2\pi i \frac{e}{m_e} A_{0X} \sum_{N=-\infty}^{\infty} T^{(N)} \delta(\hbar\omega_X + N\hbar\omega + E_1 + \Delta E_1 - E) \, . \tag{19}$$

In the last equation, in connection with the final state energy, $N > 0$ corresponds to the absorption of N photons from the laser field of frequency ω and $N < 0$ to stimulated emission. In other words, the amplitude $T^{(0)}$ corresponds to the mean peak associated to the energy $E_{\mathrm{p}h} \equiv E_1 + \Delta E_1 + \hbar\omega_X$, while $T^{(1)}$ and $T^{(-1)}$ to the satellites associated to the energies $E_{\mathrm{p}h} + \hbar\omega$ and $E_{\mathrm{p}h} - \hbar\omega$, respectively, and, so on. The truncated wavefunctions we have used allow us to consider the terms with $N = 0$, ± 1, ± 2. The other terms come out with incomplete expressions, because of the neglect of higher than two terms in the laser field amplitude, so they would not give a correct descriptions of the processes with $N > 2$.

We give the expressions for the quantities $T^{(N)}$ with $N = 0$, ± 1, ± 2,

$$\begin{aligned} T^{(0)} &= \mathcal{A}_0 \, J_0(\xi_0) + (\mathcal{A}_1 - \mathcal{A}_{-1}) \, J_1(\xi_0) \, , & (20) \\ T^{(\pm 1)} &= \mp \mathcal{A}_0 \, J_1(\xi_{\pm 1}) + \mathcal{A}_{\pm 1} J_0(\xi_{\pm 1}) \, , & (21) \\ T^{(\pm 2)} &= \mathcal{A}_0 \, J_2(\xi_{\pm 2}) \mp \mathcal{A}_{\pm 1} \, J_1(\xi_{\pm 2}) \; + \mathcal{A}_{\pm 2} \, J_0(\xi_{\pm 2}) \, . & (22) \end{aligned}$$

Because of the neglect of terms of order higher than two in the laser field amplitude, the terms include stimulated radiative corrections to the central peak ($N = 0$) only, but not to satellites ($N = \pm 1 \pm 2$). We give now the expressions of the quantities $\mathcal{A}_n$ corresponding to this approximation,

$$\begin{aligned} \mathcal{A}_0 = \mathcal{N}_1 \mathcal{N}_f \, \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle + \langle \tilde{f}_0 \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle \\ + \langle \tilde{f}_1 \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_1 \rangle + \langle \tilde{f}_{-1} \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_{-1} \rangle + \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_0 \rangle \, , \quad (23) \\ \mathcal{A}_{\pm 1} = \langle \tilde{f}_{\mp 1} \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle + \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_{\pm 1} \rangle \, , \\ \mathcal{A}_{\pm 2} = \langle \tilde{f}_{\mp 2} \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle + \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_{\pm 2} \rangle + \langle \tilde{f}_{\mp 1} \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_{\pm 1} \rangle \, . \quad (24) \end{aligned}$$

The argument (18) of the Bessel functions in (20)-(22) takes the expression

$$\xi_N = \sqrt{\frac{I_L}{I_0}} \, \frac{4}{k_L^2} \, \sqrt{k_X + N k_L - 1} \, (\mathbf{n} \cdot \mathbf{s}) \, . \tag{25}$$

We have denoted by I_0 the atomic unit of the radiation intensity ($I_0 = 3.5 \cdot 10^{16}$ W/cm^2) and by I_L the intensity of the laser radiation. The quantities $k_{X,L}$ are ratios between the photon energies and the ionization energy,

$$k_X \equiv \hbar\omega_X / \mid E_1 \mid , \qquad k_L \equiv \hbar\omega / \mid E_1 \mid \, . \tag{26}$$

We write here, for further reference that in Tzoar and Jain calculation [5] the transition amplitudes are given by

$$T_{JT}^{(N)} = (-1)^N \, \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle \, J_N(\xi_N) \, . \tag{27}$$

We prefer to transform the amplitude $T^{(0)}$ corresponding to the photoelectric peak, using a recurence relation for Bessel functions and neglecting a third-order term. Using also the relation between the vectors $\mid \tilde{f}_{\pm 1} >$ and $\mid f_{\pm 1} >$, we get

$$\begin{aligned} T^{(0)} &= [\mathcal{N}_1 \mathcal{N}_f \, \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle + \langle f_1 \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_1 \rangle + \langle f_{-1} \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_{-1} \rangle \\ &+ \langle f_0 \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle + \langle E \, \mathbf{n}- \mid \mathbf{s}_X \cdot \mathbf{P} \mid g_0 \rangle \,] \, J_0(\xi_0) \\ &+ \langle \tilde{f}_0 - f_0 \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle J_0(\xi_0) + \langle \tilde{f}_{-1} - \tilde{f}_1 \mid \mathbf{s}_X \cdot \mathbf{P} \mid E_1 \rangle J_1(\xi_0) \, . \end{aligned} \tag{28}$$

The vectors $\mid f_\nu\rangle >$ we meet now are

$$|f_{\pm 1}\rangle = \frac{\omega}{2}\,\alpha_0\, G_a(E \pm \hbar\omega)\,\mathbf{s}\cdot\mathbf{P}\mid E\,\mathbf{n}-\rangle\ , \tag{29}$$

$$|f_0\rangle = \frac{\omega^2}{4}\alpha_0^2 G_a^{\rm red}(E)\,\mathbf{s}\cdot\mathbf{P}\,[G_a(E+\hbar\omega)+G_a(E-\hbar\omega)]\mathbf{s}\cdot\mathbf{P}\mid E\,\mathbf{n}-\rangle\,. \tag{30}$$

To the first five terms in (28) one can associate seven diagrams shown in Fig. 1. The first diagram corresponds to one X-ray photon absorption and the next two describe the X-ray absorption simultaneosly with the absorption (emission) of a laser photon by the electron in the initial state and the emission (absorption) of a laser photon by the electron in the final state. The other four diagrams correspond to stimulated radiative corrections to the continuum state and to the ground state.

The first term in the (28) is the matrix element of photoeffect. All the other matrix elements in the transition amplitudes considered here can be expressed in terms of the tensors

$$\Pi_{ij}(\Omega) = \langle E\,\mathbf{n}-|P_i\,G_a(\Omega)\,P_j|E_1\rangle\,, \tag{31}$$

$$\Pi_{ijk}(\Omega',\Omega) = \langle E\,\mathbf{n}-|P_iG_a(\Omega')P_jG_a(\Omega)P_k|E_1\rangle\ , \tag{32}$$

taken for well defined values of Ω and Ω'. The tensor Π_{ij} is met in studies on K-shell Compton effect or on two-photon ionization.

The third order tensor Π_{ijk} has the structure [17, 18]:

$$\Pi_{ijk} = A\,n_i\,\delta_{jk} + B\,\left(n_j\,\delta_{ik} + n_k\,\delta_{ij}\right) + C\,n_i\,n_j\,n_k\ . \tag{33}$$

For the term in (28) containing the vector $\mid g_0 >$ the first Green's operator should be replaced with the reduced Green's operator $G_a^{\rm red}(E_1)$ attached to the ground sate.

We remark that the equations written up to now are general and could be applied for any potential $V(\mathbf{r})$.

3 The transition amplitudes in Born approximation

All that follows refers to a hydrogen-like atom with the nucleus charge Z. In this case the notation used for the Coulomb resolvent operator will be G_c (instead of G_a used in the previous section).

The Coulomb continuum solution $\mid E\,\mathbf{n}- >$ depends on one parameter,

$$\eta = \lambda/p\,, \qquad \lambda \equiv \alpha Z m_e c\,, \tag{34}$$

with α the fine-structure constant. In zeroth order Born approximation, the continuum vector $\mid E\,\mathbf{n}- >$ is replaced by a free electron solution $\mid E\,\mathbf{n} >$, which is justified if $\eta \ll 1$.

Because all the vectors $\mid \tilde{f}_\nu >$, and also $\mid f_0 >$, vanish in the Born approximation, we have to evaluate the simpler expressions,

$$\begin{aligned}\mathcal{A}_0^{\rm B} &= \langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}\mid E_1\rangle + \langle f_1^{\rm B}\mid \mathbf{s}_X\cdot\mathbf{P}\mid g_1^{\rm B} > \\ &+ \langle f_{-1}^{\rm B}\mid \mathbf{s}_X\cdot\mathbf{P}\mid g_{-1}^{\rm B} > + \langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}\mid g_0^{\rm B}\rangle\ ,\end{aligned} \tag{35}$$

$$\mathcal{A}_{\pm 1}^{\rm B} = \langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}\mid g_{\pm 1}^{\rm B}\rangle\,, \qquad \mathcal{A}_{\pm 2}^{\rm B} = \langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}\mid g_{\pm 2}^{\rm B}\rangle\,, \tag{36}$$

in order to get the transitions amplitudes (20)-(22). The index B in the last equations refers to Born approximation.

The Coulomb Green's resolvent $G_c(\Omega)$ depends on only one parameter

$$\tau = \lambda/X\ , \quad X = \sqrt{-2m_e\Omega}\,, \qquad {\rm Re}X > 0, \tag{37}$$

For $|\tau|\ll 1$ the Coulomb resolvent operator can be replaced by the free-particle resolvent $G_0(\Omega)$. We shall assume in the following that the parameters Ω and Ω' in equation (31) are such that both resolvent operators can be replaced by the free-particle resolvent, i.e., that the conditions

$$|\tau|\ll 1\,, \qquad |\tau'|\ll 1\,. \tag{38}$$

are fulfilled. We have used the notation $\tau' = \lambda/X'$, with $X' = \sqrt{-2m_e\Omega'}$, $\mathrm{R}e X' > 0$.

Working in momentum space, one gets the very simple expressions

$$< E\,\mathbf{n} \mid P_i \mid E_1 >= \frac{m_e^{1/2}}{\pi}\lambda^{5/2}\,(2p)^{3/2}\,\frac{n_i}{(p^2+\lambda^2)^2}\,, \tag{39}$$

$$\Pi^{\mathrm{B}}_{ij} = \frac{m_e^{3/2}}{\pi}\,\lambda^{5/2}\,(2p)^{5/2}\,\frac{n_i n_j}{(p^2+\lambda^2)^2\,(p^2+X^2)}\,, \tag{40}$$

$$\Pi^{\mathrm{B}}_{ijk} = \frac{m_e^{5/2}}{\pi}\,\lambda^{5/2}\,(2p)^{7/2}\frac{n_i n_j n_k}{(p^2+\lambda^2)^2(p^2+X^2)\,(p^2+X'^2)}\,. \tag{41}$$

This means, that in lowest-order Born approximation, with the notation (33) one has

$$A^{\mathrm{B}} = B^{\mathrm{B}} = 0\,, \quad C^{\mathrm{B}} = \frac{1}{\pi}\frac{(2p)^{7/2}(m_e\lambda)^{5/2}}{(p^2+\lambda^2)^2(p^2+X^2)\,(p^2+X'^2)}\,. \tag{42}$$

Using the previous expressions one obtains all the matrix-elements in (35), excepting the term in $\mathcal{A}_0$ containing the vector $\mid g_0^{\mathrm{B}}\rangle$. In next section we describe the calculation this term requires.

4 The stimulated radiative corrections to the photoionization amplitude

We consider now the amplitude of the modified one-photon absorption transcribing Eq. (28) as a more detailed expression

$$T^{(0)\,\mathrm{B}} = [\mathcal{N}_1^{\mathrm{B}}\mathcal{N}_f^{\mathrm{B}}\,\langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}\mid E_1\rangle$$
$$+\frac{2\pi\hbar\alpha}{m_e^2}\frac{I_L}{\omega^2}\Big(\langle E\,\mathbf{n}\mid \mathbf{s}\cdot\mathbf{P}G_0({\Omega_1}')\mathbf{s}_X\cdot\mathbf{P}G_0(\Omega_1)\mathbf{s}\cdot\mathbf{P}\mid E_1\rangle$$
$$+\langle E\,\mathbf{n}\mid \mathbf{s}\cdot\mathbf{P}G_0({\Omega_2}')\mathbf{s}_X\cdot\mathbf{P}\,G_0(\Omega_2)\,\mathbf{s}\cdot\mathbf{P}\mid E_1\rangle$$
$$+\,\langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}G_c^{\mathrm{red}}(E_1)\,\mathbf{s}\cdot\mathbf{P}\,[G_0(\Omega_1)+G_0(\Omega_2)]\mathbf{s}\cdot\mathbf{P}\mid E_1\rangle\Big)]\,J_0(\xi_0) \tag{43}$$

In front of the corrections one has a factor proportional to the laser intensity I_L. The parameters met in the Green's operator of the free particle are

$$\Omega_1{}' = E+\hbar\omega\,,\quad \Omega_1 = E_1+\hbar\omega\qquad \Omega_2{}' = E-\hbar\omega\,,\quad \Omega_2 = E_1-\hbar\omega\,. \tag{44}$$

As shown in Appendix A, consistent with the zeroth order Born approximation, one has $\mathcal{N}_1^{\mathrm{B}} = 1-4I_L/(3I_0\,k_L^4)$, and the continuum normalization constant $\mathcal{N}_f^{\mathrm{B}}$ is 1.

We notice that in zeroth order Born approximation the diagrams 4 and 5 from Fig.1 give no contribution. In fact, there is a cancellation of singular terms coming from each of these two diagrams.

In the last term in (43),

$$\mathcal{P}^{\mathrm{B}} \equiv \langle E\,\mathbf{n}\mid \mathbf{s}_X\cdot\mathbf{P}G_c^{\mathrm{red}}(E_1)\,\mathbf{s}\cdot\mathbf{P}\,[G_0(\Omega_1)+G_0(\Omega_2)]\mathbf{s}\cdot\mathbf{P}\mid E_1\rangle\,, \tag{45}$$

we have to use the reduced Coulomb Green operator. The analytic evaluation of this term, a laborious task, is briefly described in Appendix B. The result for $\mathcal{P}^{\mathrm{B}}$ is

$$\mathcal{P}^{\mathrm{B}} = -\frac{8\sqrt{2}}{\pi}\left(1+2\frac{k_L^2+k_X}{k_X^2-k_L^2}\mid\mathbf{n}\cdot\mathbf{s}\mid^2\right)\frac{\mathbf{n}\cdot\mathbf{s}_X}{k_L^2k_X^{5/4}} \tag{46}$$

Then, the final result for the transition amplitude (43) is

$$T^{(0)\,\mathrm{B}} = T_{\mathrm{PT}}^{(0)}\,J_0(\xi_0)\;, \tag{47}$$

$$T_{\mathrm{PT}}^{(0)} = \left[1-\frac{8}{3}\,\frac{I_L}{I_0}\,\frac{1}{k_L^4}\left(2+3\,\frac{k_L^2k_X+k_L^2+k_X}{k_X^2-k_L^2}\quad\mid\mathbf{n}\cdot\mathbf{s}\mid^2\right)\right]\,F_0^{\mathrm{B}}(\mathbf{n}\cdot\mathbf{s}_X) \tag{48}$$

where

$$F_0^{\mathrm{B}} = \frac{2\,\sqrt{2}}{\pi\,(k_X-1)^{5/4}}\,. \tag{49}$$

In perturbatiom theory, in lowest order in αZ, the result will be $T_{\mathrm{PT}}^{(0)}$ with the first term in the bracket being $1-\xi_0^2/4$, instead of 1.

5 Numerical results for the assisted K-shell photoeffect

The differential cross section (DCS) of the process in which N photons of frequency ω are exchanged, simultaneously with the absorption of the
X-photon, is connected with the transition amplitude $T^{(N)}$ by

$$\frac{d\sigma^{(N)}}{d\Omega} = \frac{8\pi^2\alpha}{k_X}\,a_0^2\,\mid T^{(N)}\mid^2\,, \tag{50}$$

where a_0 is the Bohr radius.

Before giving numerical results, we mention the simple expressions of the amplitudes for the satellites, valid if dressing effects are neglected . These coincide with the results in perturbation theory and lowest order in αZ, namely,

$$T_{\mathrm{P}T}^{(\pm 1)} = 2\sqrt{\frac{I_L}{I_0}}\,(k_X-1)^{5/4}\frac{k_X \pm k_L}{(k_X \pm k_L - 1)^{3/4}\,k_X\,k_L}\,F_0^{\mathrm{B}}(\mathbf{n}\cdot\mathbf{s})\,, \tag{51}$$

$$T_{\mathrm{P}T}^{(\pm 2)} = 2\,\frac{I_L}{I_0}\,(k_X-1)^{5/4}\frac{k_X \pm 2k_L}{(k_X \pm 2k_L - 1)^{1/4}\,k_X\,k_L^4}\,F_0^{\mathrm{B}}(\mathbf{n}\cdot\mathbf{s})^2(\mathbf{n}\cdot\mathbf{s}_X)\,, \tag{52}$$

with F_0 defined in (49).

In Tzoar-Jain approximation (27) the results are, for any N,

$$T_{JT}^{(N)} = (-1)^N\,F_0^{\mathrm{B}}(\mathbf{n}\cdot\mathbf{s}_X)\;J_N(\xi_N)\,. \tag{53}$$

In order to have some check of our calculation, we have made a comparison with numerical results, given as tables by Klarsfeld and Maquet [20], for the ionization by absorption of three identical photons, of energy k_L defined as in (26). The expression we use for the generalized cross-section (three-photon absorption cross-section divided by the square of intensity) comes from the amplitude $T_{\mathrm{P}T}^{(2)}$ in (52) and it is simply

$$\frac{\sigma^{\mathrm{L},C}}{I_L^2} = \frac{2^{10}\pi\alpha}{7}\,\frac{\beta_{\mathrm{L},C}}{k_L^9\sqrt{3k_L-1}}\left(\frac{a_0}{I_0}\right)^2\,, \tag{54}$$

with $\beta_{\mathrm{L}} = 1$, $\beta_{\mathrm{C}} = 2/5$. In figure 2 we show our results for the generalized cross-section for linear and circular polarizations of photons, compared with the results of Klarsfeld and Maquet, in the domain of wavelengths ($50\div 850$) Å . We find a good agreement, although in the domain of wavelengths greater than 500 Å the parameters $\mid \tau \mid$ and $\mid \tau' \mid$ of the Coulomb Green's functions involved, which are required by our approximation to be much smaller than 1, are greater than 1.

The numerical results presented in Figures 3 and 4 refer to the two radiation beams having linear parallel polarizations $\mathbf{s}_X = \mathbf{s}_L$ taken along the $0z$ axis of the coordinate system.

In Fig. 3 we compare our results with those of Tzoar and Jain approximation, based on Eq. (53), for $\lambda_L = 500$ Å . θ is the angle between the photoelectron momentum and the polarization direction, the same for the two beams.The full curve corresponds to $\lambda_X = 490$ Å and $I_L = 10^{13}\mathrm{W/cm}^2$, while the dashed curve to $\lambda_X = 450$ Å and $I_L = 10^{14}\mathrm{W/cm}^2$. The deviation of the plotted ratio from 1 comes from the second-order corrections in the laser field amplitude, not included in previous calculations. In this high frequencies regime the differences are small, and they can be increased only by increasing the intensity or/and decreasing the gap between the two frequencies.

In Fig.4 we refer also to angular distributions. We represent the differential cross section $d\sigma^{(N)}/d\Omega$ for the same conditions as in Fig. 3. Panels (a), (b) and (c) correspond, respectively, to $N = 0$, 1, and 2. There are order of magnitude differences between the three cases. The situation would change at lower frequencies, beyond the regime of validity of the equations used in the present calculation. For $N = -1$ and $N = -2$ the distributions are close to those corresponding to $N = 1$ and $N = 2$, respectively.

6 Conclusions

We have considered the modification of the K-shell photoionization due to the presence of a second source of electromagnetic radiation of moderate intensity. The equations (20)-(22) and (23) include laser stimulated radiative corrections to the one-photon absorption amplitude. The analytic evaluation of these corrections was done in the lowest order in αZ. This is a preliminary study, which will be followed by calculations including Coulomb field effects coming from the Green's operator and the final state of the electron, allowing numerical evaluations at lower frequencies.

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Appendix A: The normalization constants $\mathcal{N}_1$ and $\mathcal{N}_f$

The normalization condition, $\langle \Psi_1 \mid \Psi_1 \rangle = 1$, for the wavevector given in Eqs. (8) and (11) leads to

$$\mathcal{N}_1 = 1 - \frac{1}{2}[\,\langle g_1 \mid g_1 \rangle + \langle g_{-1} \mid g_{-1} \rangle\,]\,, \tag{55}$$

with the neglect of terms of order higher than two in the parameter α_0 given by (10).

For the ground state of a hydrogenlike atom the scalar products in (55) have exact analytic expressions. For consistency with the whole calculation, we consider them here only in Born approximation,

$$\langle g^{\mathrm{B}}_{\pm 1} \mid g^{\mathrm{B}}_{\pm 1} \rangle = \frac{\omega^2\,\alpha_0^2}{4} \langle E_1 \mid \mathbf{s}\cdot\mathbf{P}\, G_0(\Omega_\pm)\, G_0(\Omega_\pm)\, \mathbf{s}\cdot\mathbf{P} \mid E_1 \rangle\,, \qquad \Omega_\pm = E_1 \pm \hbar\omega\,, \tag{56}$$

i.e., for the Coulomb resolvent replaced with the free particle resolvent G_0. The result,

$$\langle g^{\mathrm{B}}_{\pm 1} \mid g^{\mathrm{B}}_{\pm 1} \rangle = \frac{\lambda^4}{3m_e^2} \frac{I_L}{I_0} \frac{1}{k_L^2\, \Omega_\pm^2}\,, \tag{57}$$

leads to

$$\mathcal{N}_1^{\mathrm{B}} = 1 - \frac{4}{3}\frac{I_L}{I_0}\frac{1}{k_L^4}\,. \tag{58}$$

In the continuum case, we impose the condition $\langle \Psi_f(\mathbf{p},t) \mid \Psi_f(\mathbf{p}',t) \rangle = \delta(E-E')\,\delta(\mathbf{n}-\mathbf{n}')$ to the vectors given by (16) for different values of the asymptotic momentum $\mathbf{p}$. Because in zeroth order Born approximation the corrections to the Volkov solutions vanish, we have $\mathcal{N}_f^{\mathrm{B}} = 1$.

Appendix B: The contribution of the stimulated radiative correction to the ground state

The matrix element $\mathcal{P}^{\mathrm{B}}$ in (45) needs the evaluation of the tensor

$$\Pi^{\mathrm{red\,B}}_{ijk}(\Omega) \equiv \langle E\,\mathbf{n}\,| P_i G_c^{red}(E_1) P_j G_0(\Omega) P_k | E_1 \rangle\ . \tag{59}$$

As the Coulomb Green's function has a compact analytic expression, we prefer to start from the tensor

$$\overline{\Pi}_{ijk}(\Omega', \Omega) = \langle E\,\mathbf{n}\,| P_i G_c(\Omega') P_j G_0(\Omega) P_k | E_1 \rangle\ , \tag{60}$$

which has a singularity for $\Omega' \to E_1$, due to the contribution of the ground state to the Green's operator $G_c(\Omega')$. The singular term is

$$p^{\mathrm{B}}_{ijk}(\Omega', \Omega) = \frac{< E\,\mathbf{n} \mid P_i \mid E_1 >< E_1 \mid P_j G_0(\Omega) P_k \mid E_1 >}{\Omega' - E_1} \tag{61}$$

and has the simple analytic expression

$$p^{\mathrm{B}}_{ijk}(\Omega', \Omega) = \frac{\tau'^2}{1-\tau'^2} A^{\mathrm{B}}_s\,, \qquad A^{\mathrm{B}}_s = \frac{8\sqrt{2}\, m_e^{5/2}}{3\pi X^2} \eta^{5/2}\,, \tag{62}$$

with η, X and τ' defined in Sect. 3.

Then, we evaluate the tensor $\Pi^{\mathrm{red\,B}}_{ijk}(\Omega)$, defined in (59), as

$$\Pi^{\mathrm{red\,B}}_{ijk}(\Omega) = \lim_{\Omega' \to E_1} [\overline{\Pi}_{ijk}(\Omega', \Omega) - p^{\mathrm{B}}_{ijk}(\Omega', \Omega)\,]\,. \tag{63}$$

Working in momentum representation, we find easily that

$$\overline{\Pi}_{ijk}(\Omega', \Omega) = -\frac{1}{\pi}(2\lambda)^{5/2}(pm_e)^{3/2} n_i D_{jk}\,, \tag{64}$$

where

$$D_{jk} = \int_{\mathbf{q}} G_c(\mathbf{p}, \mathbf{q}; \Omega')\, \frac{q_j q_k}{(q^2+\lambda^2)^2 (q^2+X^2)}\, d\mathbf{q}\,. \tag{65}$$

The integral D_{jk} is built from two scalar functions, $\mathcal{D}_1$ and $\mathcal{D}_2$, as

$$D_{jk} = \mathcal{D}_1\, \delta_{jk} + \mathcal{D}_2\, n_j n_k\,. \tag{66}$$

We found convenient to evaluate $\mathcal{D}_1$ and the linear combination

$$D \equiv 3\,\mathcal{D}_1 + \mathcal{D}_2 = \int_{\mathbf{q}} G_c(\mathbf{p}, \mathbf{q}; \Omega')\, \frac{q^2}{q^2+X^2}\, \frac{d\mathbf{q}}{(q^2+\lambda^2)^2}\,, \tag{67}$$

The triple integrals in $\mathcal{D}_1$ and D can be performed, if we use Schwinger representation [19] of the Coulomb's Green function G_c in momentum space. After that, we are left with a one-dimensional integral coming from the Schwinger formula. During the calculation we have in mind that we have to extract the singular term for $\tau' \to 1$ and to obtain correctly the finite term. This means that τ' can be replaced by 1 in the terms which do not contain the singularity at $\tau' = 1$.

Without giving more details on these tedious calculations, we write the final results for $\mathcal{D}_1$ and D,

$$\mathcal{D}_1 \;=\; -\frac{2m_e}{3p^4 X^2}\left(\frac{\tau'^2}{1-\tau'^2} + \frac{3}{2}\right) + \mathcal{O}(1-\tau')\,, \tag{68}$$

$$D \;=\; -\frac{m_e}{p^4 X^2}\left(\frac{2\tau'^2}{1-\tau'^2} + \frac{3p^2+5X^2}{p^2+X^2}\right) + \mathcal{O}(1-\tau')\,, \quad \mid X \mid \ll \lambda\,. \tag{69}$$

From this one derives for the tensor (60)

$$\overline{\Pi}_{ijk}(\Omega', \Omega) = \overline{A}\, n_i\, \delta_{jk} + \overline{C}\, n_i\, n_j\, n_k \tag{70}$$

with

$$\overline{A} = \frac{8\sqrt{2} m_e^{5/2}}{3\pi\, X^2}\left(\frac{\lambda}{p}\right)^{5/2}\left(\frac{\tau'^2}{1-\tau'^2} + \frac{3}{2}\right)\,, \qquad \overline{C} = \frac{8\sqrt{2} m_e^{5/2}}{3\pi}\left(\frac{\lambda}{p}\right)^{5/2} \frac{1}{p^2+X^2}\,. \tag{71}$$

As it should be, the first term is identical with the contribution of the ground state, given in (62). Finally, for the tensor (59) one has

$$\Pi_{ijk}^{\mathrm{red\,B}}(\Omega) = A^{\mathrm{red\,B}}\, n_i\, \delta_{jk} + C^{\mathrm{red\,B}}\, n_i\, n_j\, n_k\,, \tag{72}$$

where

$$A^{\mathrm{red\,B}} = \frac{4\sqrt{2}m_e^{5/2}}{\pi X^2}\left(\frac{\lambda}{p}\right)^{5/2}, \qquad C^{\mathrm{red\,B}} = \frac{2X^2}{p^2+X^2}\, A^{\mathrm{red\,B}}\,. \tag{73}$$

The calculations of the triple integrals on $\mathbf{q}$ makes use of two integrals,

$$\int_{\mathbf{q}} \frac{d\mathbf{q}}{(q^2+a^2)(q^2+b^2)} = \frac{2\pi^2}{a+b}\,, \qquad \mathrm{Re}\, a > 0\,,\ \mathrm{Re}\, b > 0\,, \tag{74}$$

$$\int_{\mathbf{q}} \frac{d\mathbf{q}}{[(\mathbf{q}-\mathbf{p})^2+a^2]\,(q^2+b^2)\,(q^2+c^2)} = \frac{i\,\pi^2}{V}\ln\frac{U-iV}{U+iV}\,, \tag{75}$$

with

$$\begin{aligned} U &= (b+c)\,[\,(a+b)(a+c)+p^2\,]\,, \\ V &= p\,(b^2-c^2)\,, \qquad\qquad \mathrm{Re}\, a > 0\,,\ \mathrm{Re}\, b > 0\,, \mathrm{Re}\, c > 0\,, \end{aligned} \tag{76}$$

and of other integrals which can be obtained from them by derivation with respect to the parameters on which these integrals depend.

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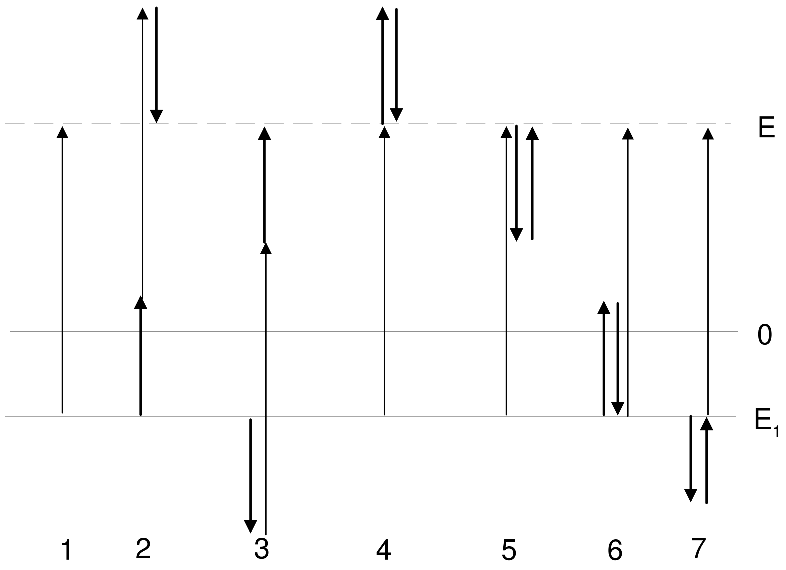

Fig.1 Diagrams associated to one-photon ionization from the ground state, including stimulated radiative corrections due to the simultaneous interaction with a laser field.

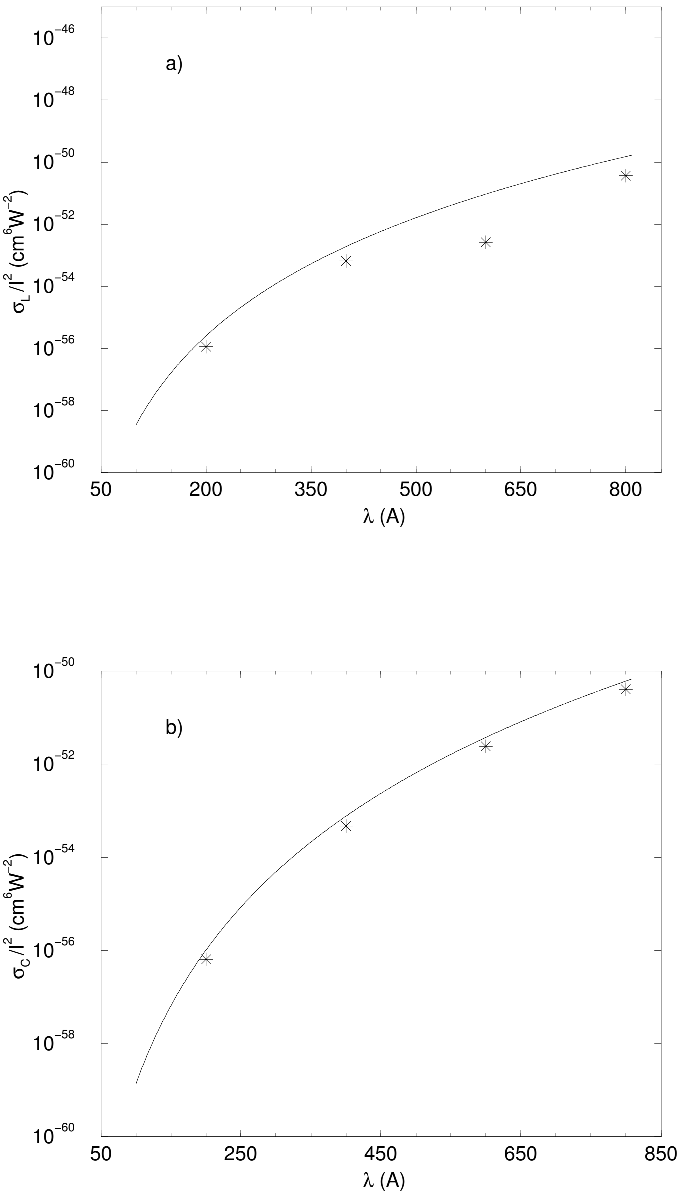

Fig.2 The generalized cross section (54), σ/I^2 (in cm^6 W^{-2}), for three-photon ionization from ground state atomic hydrogen, as function of the wavelength λ (Å): (a) linear polarization; (b) circular polarization. The stars denote the results reported by Klarsfeld and Maquet [20].

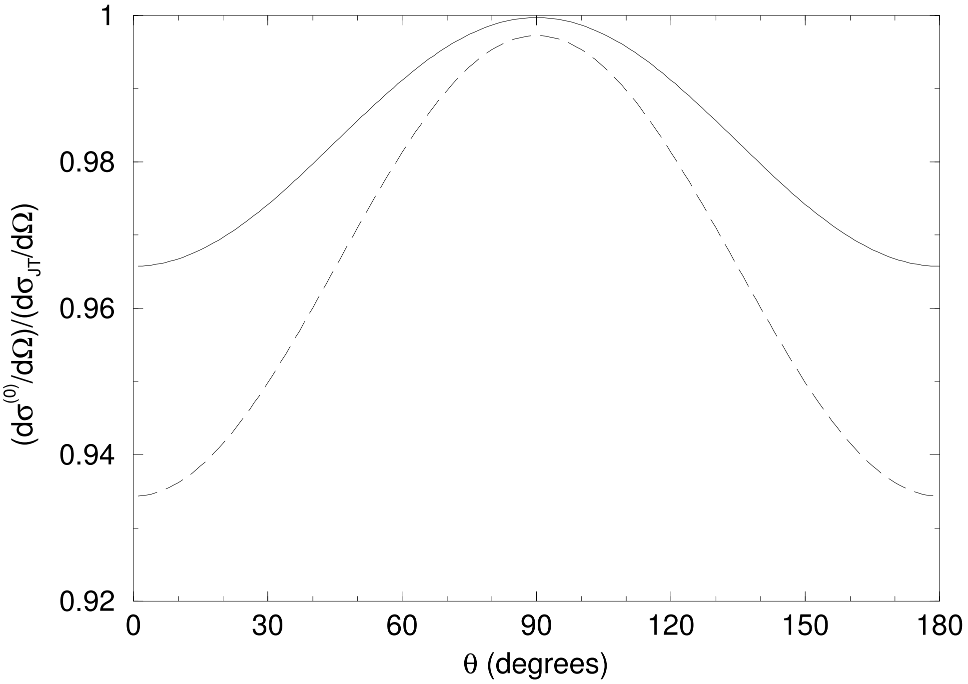

Fig.3 The differential cross section $d\sigma^{(0)}/d\Omega$, divided by $d\sigma_{JT}^{(0)}/d\Omega$ corresponding to Tzoar and Jain approximation, as a function of the polar angle θ for $\lambda_L = 500$ Å. The full line corresponds to $\lambda_X = 490$ Å and $I_L = 10^{13}$ W/cm^2, the dashed line to $\lambda_X = 450$ Å and $I_L = 10^{14}$ W/cm^2.

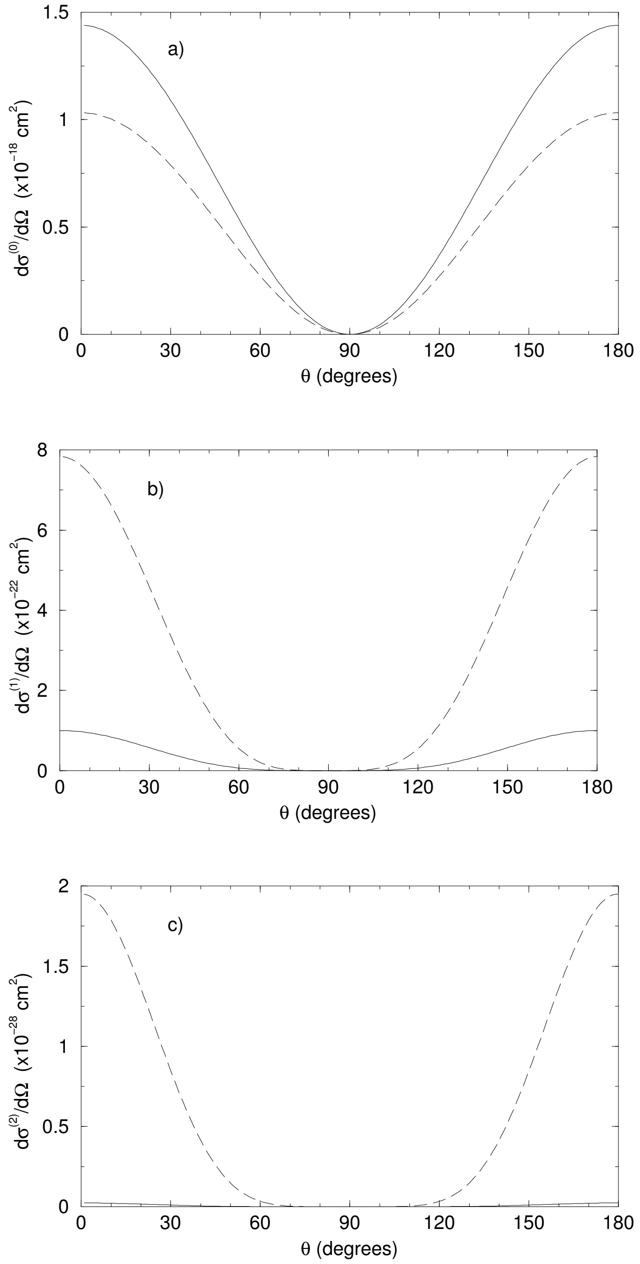

Fig. 4 The dependence of the differential cross section $d\sigma^{(N)}/d\Omega$ on the polar angle θ in the same conditions as in Fig. 3; (a) N=0, (b) N=1, (c) N=2.