Estimates of the emission rates and "fine structure" intensity patterns for deformed nuclei

I. Silişteanu[1], W. Scheid[2], M. Rizea[1], and A.O. Silişteanu[1]
[1]Institute of Atomic Physics, Bucharest-Magurele, Romania
[2]Institut für Theoretische Physik der Justus-Liebig-Universität, Giessen, Germany

Abstract

We will review some of the recent advances in both structure and dynamics of heavy nuclei focussing on the issues currently of interest along with possible direct applications to resonant particle spectroscopy technique in next experimental studies of clustering and decaying phenomena. We propose a model, theoretical and mathematical formalism and computational methods necessary for low energy resonant scattering analysis of deformed nuclei and show how the model works for α-decay. Extensions and applications of the model to new phenomena like subbarrier fusion, quasifission, formation of nuclear molecules, decay of dinuclear system are also discussed.

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1 Introduction

In present work we propose an approach to reduce the problem of particle emission rates of spheroidal nuclei to the solution of homogeneous and inhomogeneous systems of coupled differential equations arising from the Schrodinger equation. We show that some of the previous development in the question carried out so far in the Wigner R-matrix formalism can be restated in a slightly different form within the Feshbach formalism of resonance nuclear reactions which has advantages of unified treatment of the structure and nuclear dynamics and of emphasis on the more essential mathematical steps. Additionally, the treatment of the decay rates for radioactive states associated with resonances in deformed field is more general and inherently less dependent on arbitrary assumptions than other available R-matrix treatments.

In this paper, we develop new theoretical and computing methods for obtaining the particle emission rates taking into the account of non-sphericity of interaction which complicates the solution of the differential equations describing the "fine structure" intensity patterns in multichannel decay. First of all, we develop a procedure to iterate the nuclear interaction directly in equations of motion in order to obtain the solution for resonant multichannel decay of ground or excited radioactive states. We calculate the emission rates and "fine structure" intensity patterns and confront these to data and results of other models. Some predictions are made for the αhalf-lives of possible new deformed superheavy elements (SHE) $[1-4]$. The procedure may be useful for designing the resonant particle decay spectroscopy technique in next experimental studies of clustering and decaying phenomena involving the radioactive states.

The solution of coupled equations is obtained by a direct numerical integration using step-by-step methods on computer. However, in the case of large number of channels the solution remains a very difficult and time-consuming operation, and therefore it is of major interest to find the most efficient methods of integration. Some variants of the Gordon method are found to be most efficient for these problems.
In Section 2 we present the basic formulas for calculating the emission rates in the single channel and many channel cases. In Section 2.1 we stress some of the resonance features of the single channel solution that we shall require for the extension, in Section 2.2 to the resonance solution in many channel case. Applications of the model and concluding remarks are presented in Sect. 3.

2 An outline of the theory

2.1 Single channel

In the case of α-decay of a single resonance state ν into a channel of angular momentum l, the partial decay width is given by [5, 6]:

$$\Gamma_l^\nu = 2\pi \left| \frac{\int_{r_{\min}}^{r_{\max}} I_l^\nu(r) u_l^0(r) dr}{\int_{r_{\min}}^{r_{\max}} I_l^\nu(r) u_l^\nu(r) dr} \right|^2 \tag{1}$$

where $I_l^\nu(r)$ is the particle (cluster) formation amplitude (FA) and $u_l^\nu(r)$ and $u_l^0(r)$ are the solutions of the system of differential equations

$$\left[\frac{d^2}{dr^2} + k^2(r) - \frac{l(l+1)}{r^2} - \frac{2m}{\hbar^2}\, V(r)\right] u_l^0(r) = 0 \tag{2}$$

$$\left[\frac{d^2}{dr^2} + k^2(r) - \frac{l(l+1)}{r^2} - \frac{2m}{\hbar^2} V(r)\right] u_l^\nu(r) = \frac{2m}{\hbar^2} I_l^\nu\ (r) \tag{3}$$

The boundary conditions for $u_l^\nu(r)$ and $u_l^0(r)$ are:

$$u_l^{0(\nu)}(r = 0) = 0 \tag{4}$$

$$u_l^0(r \to \infty) \to u_l^{(-)}(r) - e^{2i\delta} u_l^{(+)}(r), u_l^\nu(r \to \infty) \to 0 \tag{5}$$

where δ is the phase shift, $u_l^{(\pm)}(r) = G_l(r) \pm iF_l(r)$ and F_l and G_l are the regular and irregular Coulomb functions.
Eqs.(2) and (3) describe the radial motion of the fragments at large and small separations in terms of the wave number k ($k^2 = (2m/\ \hbar^2)Q$, reduced mass m, kinetic energy Q of emitted particle (in the center-of-mass system), inter-fragment interaction $V(r)$, and FA. The potential includes the nuclear and Coulomb spherical terms (see Appendix):

$$V(r)\ = V^{nucl.}(r) + V^{coul.}(r). \tag{6}$$

The FA is the antisymmetrized projection of the parent wave function (WF) $\mid \Psi_\nu >$ on the channel WF $\mid c\rangle =\mid [\Phi_1(\eta_1)\Phi_2(\eta_2)Y_{lm}(\hat{r})]_c\rangle$:

$$I_c^\nu(r) = r\langle \Psi_\nu \mid \mathcal{A}\{[\Phi_1(\eta_1)\Phi_2(\eta_2)Y_{lm}(\hat{r})]_c\}\rangle. \tag{7}$$

where $\Phi_1(\eta_1)$ and $\Phi_2(\eta_2)$ are the internal wave functions of the fragments, $Y_{lm}(\hat{r})$ is the wave function of the angular motion, $\mathcal{A}$ is the inter-fragment antisymmetrizer, r connects the centres of mass of the fragments, and the symbol $<|>$ means integration over the internal coordinates and angular coordinates of relative motion. The usual methods in calculation of the FA are reviewed in Appendix.

For the lower limit in integrals (1) we choose an arbitrary small radius $r_{\min} > 0$. The upper limit $r_{\max}$ is close to the first exterior node of $u_l^0(r)$ since this function decreases rapidly with r after the barrier and is essentially confined in the interval $[r_{\min}, r_{\max}]$.

Since Eq.(2) is linear and homogeneous any solution with the correct behaviour at the origin will be a multiple of any other solution with the same initial condition. Thus the second point in the solution may be chosen completely arbitrarily and the overall normalization determined by matching to the required form in the asymptotic region.

To avoid the usual ambiguities encountered in formulating the potential for the resonance tunnelling of the barrier we iterate directly optical potential in equations of motion . The "one-body" (o.b.) resonance width given in terms of eigenvalues and eigenfunctions of the system is:

$$\Gamma_l^{o.b.} = 2\pi \left| \frac{\int_{r_{\min}}^{r_{\max}} u_l^0(r) u_l^0(r) dr}{\int_{r_{\min}}^{r_{\max}} u_l^0(r) u_l^{o.b.}(r) dr} \right|^2 \tag{8}$$

where $u_l^{o.b.}(r)$ is a solution of Eq.(2) in which $I_c^\nu(r)$ is merely replaced by $u_l^0(r)$.

2.2 Coupled channels

The experimental signatures of radioactive states in nuclei have traditionally been strong and can be supported by selective excitation in α-transfer reactions, vibrationally or rotationally spaced energy levels, enhanced electromagnetic moments and transition strengths, and appreciable cluster particle width for resonant states above the decay treshold. First of all, the experimental observation of α-particle groups with similar energies suggests that the parent and daughter nuclei are deformed. This may reveal important information on interplay between nuclear structure and dynamics and provides in many cases a powerful basis for extending the spin-parity information from the daughter nuclide to the unknown parent. Let us consider the case when exist appreciable interactions between the emitted particle waves with different final states of the residual nucleus. For the particle emission from a decaying state of the total spin I into a set of decay channels c (defined the total spin of residual nucleus R and the orbital momentum l of particle, $c = \{R,l\}$), the following two systems of coupled differential equations is obtained (see Appendix):

$$\left[\frac{d^2}{dr^2} + k_{Rl}^2(r) - \frac{l(l+1)}{r^2} - \frac{2m}{\hbar^2} V^{(0)}(r)\right] u_{Rl}^0(r)$$

$$-\frac{2m}{\hbar^2} \beta V^{(2)}(r) \sum_{R'l' \neq Rl} W_{RlR'l'} u_{R'l'}^0(r) = 0 \tag{9}$$

$$\left[\frac{d^2}{dr^2} + k_{Rl}^2(r) - \frac{l(l+1)}{r^2} - \frac{2m}{\hbar^2} V^{(0)}(r)\right] u_{Rl}^\nu(r)$$

$$-\frac{2m}{\hbar^2} \beta V^{(2)}(r) \sum_{R'l' \neq Rl} W_{RlR'l'} u_{R'l'}^\nu(r) = \frac{2m}{\hbar^2} I_{Rl}^\nu\ (r) \tag{10}$$

where, $k_{Rl}^2 = E_I - E_R - E_l$, $I = R + l$, and $E_I - E_R - E_l$.are the total energies of initial (E_I) and final nuclei (E_R,E_l).

We restrict attention to the case in which all decay channels are open so that the boundary conditions are

$$u^0_{Rl}(r=0)=0, u^\nu_{Rl}(r=0)=0. \tag{11}$$

$$u^0_{Rl}(r \longrightarrow \infty) \sim \delta_{RR'}\delta_{ll'}\exp\left[-i(k_{RR'}r-l\pi/2)\right]-(k_{R'}/k_R)^{1/2}S^J(Rl,R^{'}l^{'}) \exp\left[+i(k_{RR'}r-l^{'}\pi/2)\right],$$

$$u^\nu_{Rl}(r \longrightarrow \infty)=0. \tag{12}$$

where S is the scattering matrix. If the matrix elements of the interaction potential have no singularities of order two or higher at the origin, then for small r the solutions of (9) with that satisfy (11) are given by $u^0_{Rl}(r\ )=a_{Rl}r^{l+1}$where a is a matrix of constants. The solutions thus obtained will not, in general, satisfy the asymptotic boundary conditions (12). Thus, N linearly independent solutions of (9) must be found and a suitable linear combination of them matched to the correct asymptotic form. The solutions $u^0_{Rl}(r\ )$may be matched to the boundary conditions at two values of r large enough so the terms $V_{RlRl'}$are negligible (see Appendix). A special type of eigenvalue solution will be considered here for which the behaviour of solution in each separate channel is similar to that of G_l in the one channel problem. In each channel the absolute value of $u^0_{Rl}(r\ )$decreases to the of small fraction of its value inside of nucleus and only after that enters the region within which it has an oscillatory character (the condition being similar to that of resonances in the central field). Such a solution can be modified by introduction of potential barrier extending to large r in order to convert the decaying state into a discrete level with a single decay channel. The solution inside the nucleus can be expected to be unaffected by the removal of the barriers and its amplitude to decreases approximately with the time as in the case of one channel problem. The partial width in the channel (Rl) is

$$\Gamma^\nu_{Rl}=2\pi\left|\frac{\int_{r_{\min}}^{r_{\max}} I^\nu_{Rl}(r)u^0_{Rl}(r)dr}{\int_{r_{\min}}^{r_{\max}} I^\nu_{Rl}(r)u^\nu_{Rl}(r)dr}\right|^2 \tag{13}$$

Assuming that the FA can be simply factorized [6]as:

$$I^\nu_{Rl}\ (r)=(S^\nu_{Rl})^{1/2}G_l(r) \tag{14}$$

($S^\nu_{Rl}$is the spectroscopic factor), the width (13) can be rewritten as:

$$\Gamma^\nu_{Rl}=S^\nu_{Rl}\Gamma^{o.b.}_{Rl} \tag{15}$$

where the o.b. resonance width is:

$$\Gamma^{o.b.}_{Rl}=2\pi\left|\frac{\int_{r_{\min}}^{r_{\max}} G_l(r)u^0_{Rl}(r)dr}{\int_{r_{\min}}^{r_{\max}} G_l(r)u^{o.b.}_{Rl}(r)dr}\right|^2. \tag{16}$$

$u^{o.b.}_{Rl}$being a solution of (10) in which the inhomogeneity is merely $G_l(r)$.
In form the result (16) is like the one $\Gamma_{Rl}=\gamma^2_l(r=R_c)P_{Rl}(r=R_c)$ known in the R-matrix theory. While this width is estimated at a somewhat arbitrary channel radius $R_c$this radius is not necessary in estimation (16).

3 Conclusions

Theoretical method described in Sect 2 may be useful for designing the resonant particle decay spectroscopy techniques in next experimental studies of clustering and decaying phenomena involving the radioactive states of deformed SHE. In the future work, our group plan to apply this method to:
- investigate the effects of nuclear deformation on the decay rates in long α-decay chains of SHE;
- study the dependence of fission barriers on angular momentum and deformation;
- explore in detail the radioactive ground and excited states of new SHE $[1-4]$;
- estimate microscopically the decay rates, electromagnetic moments and branching ratios and compare the results with the ones of $[7-9]$.
- extend the developments $[10-12]$ to exotic nuclei and new phenomena.

References

[1] S. Hofmann and G. Münzenberg, Rev. Mod. Phys. 72,733(2000).

[2] S. Hofmann et al., Eur.Phys.J. A 10, 5 (2001).

[3] Yu. Ts. Oganessian et al., Phys. Rev. Lett 83,3154(1999);

[4] Yu.Ts. Oganessian et al., Phys.Rev. C63, 011301 (2001).

[5] I. Silişteanu, W. Scheid, Phys.Rev. C 51,2023(1995).

[6] I. Silişteanu, W. Scheid and A. Sandulescu, Nucl. Phys A 679, 317(2001).

[7] P. Möller, J.R. Nix and K.-L. Kratz, At. Data and Nucl. Data Tab. 66,131(1997).

[8] Z. Patyk and A. Sobiczewski, Nucl. Phys. A354, 229(1996).

[9] K. Rutz, M. Bender, T. Burvenich, T. Schilling, P.-G. Reinhardt, J.A. Maruhn, W. Greiner, Phys. Rev. C 56, 238(1997).

[10] M.Ivascu and I.Silisteanu, Nucl.Phys.A 485,93(1988).

[11] I. Silisteanu and M. Ivascu, J. Phys. G: Nucl. Part. Phys. 15(1989)1405.

[12] M.Ivascu and I.Silisteanu, Sov.J. Part.Nucl. 21,589(1990).

4 Appendix

4.1 Interaction potential

Let us consider the following radial shape for the residual nucleus (RN):

$$R(\theta,\phi) = R_0(1 + Y_2(\theta,\phi)) \tag{17}$$

where $(\theta,\phi)$are the angular coordinates referred to the body fixed system; $\beta$is the quadrupole deformation parameter. Assuming that the particle-RN interaction depends on the relative angular coordinates $\hat{r}$referred to a space fixed system and the internal coordinates $\xi$of the RN:

$$V(r,\xi) = V[r - R(\theta,\phi)] \tag{18}$$

we obtain for the first order expansion of (18) in powers of β:

$$V[r-R(\theta,\phi)] = V^{(0)}(r-R_0) + \beta V^{(2)}(r-R_0)\sum_m (4\pi/5)^{1/2} Y_{2m}(\hat{r})Y_{2m}(\eta_1), \tag{19}$$

where $R_0 = r_0 A^{1/3}$. Hereafter the following expression are assumed for $V^{(0)}(r-R_0)$and $V^{(2)}(r-R_0)$

$$V^{(0)}(r-R_0) = -V_0 f_a(r) - iWg_b(r) + V_{so}2(\hbar/m_\pi c)^2 \frac{1}{r}\frac{df_a}{dr}(l*s) \tag{20}$$

$$V^{(2)}(r-R_0) = -V_0 R_0 \frac{df_a}{dr} \tag{21}$$

where

$$f_a(r) = [1+\exp(r-R_0)/a] \tag{22}$$

$$g_b(r) = 4\exp(r-R_0)/af_b^2(r) \tag{23}$$

where $a$and $b$are diffuseness parameters. For an attractive potential V_0(depth) and V_{so}(spin-orbit) are parameters. The matrix elements of the interaction potential are given by :

$$< R,l;J \mid V \mid R',l';J >= \delta_{RR'}\delta_{ll'}V_{Rl}(r) + W_2(R,l,R',l';J)V_2(r), \tag{24}$$

the W_2coefficients being evaluated from formulas given by[6].

4.2 Equations of motion

For the emission of spinless particle ($s=0$) the total angular momentum $J$is related to the spin of residual nucleus $R$and the relative orbital momentum $l$of the particle by

$$J = R + l. \tag{25}$$

The Hamiltonian of the system is

$$H = T_r + V[r-R(\theta,\phi)] + H_1 + H_2, \tag{26}$$

where $T_r$is the kinetic energy operator for the relative motion and $H_1$and $H_2$are the fragment Hamiltonians.
For the RN we assume the Hamiltonian of a rigid rotator with eigenfunctions $\Psi_R^{m_R}$given by:

$$H_1\Phi_1^{Rm_R} = E_1\Psi_1^{Rm_R} \tag{27}$$

with

$$\Phi_1^{Rm_R} = [(2R+1)/8\pi^2]D_{m_R 0}^{R*}(\alpha\beta\gamma) \tag{28}$$

and the phase convention:

$$D_{m_R 0}^{R}(\alpha\beta\gamma) = [4\pi/(2R+1)]Y_R^{m_R *}(\beta\alpha). \tag{29}$$

The eigenstates of the emitted particle are given by

$$H_2\Phi_2 = E_2\Phi_2 \tag{30}$$

We write the total scattering wave function of the system as follows:

$$| \chi_E >= \sum_{JM} \chi_E^{JM}(r, \eta_1, \eta_2) = \sum_{JM} \frac{1}{r} \sum_{Rjl} u_{Rjl}^J(r) \Phi_{Rjl}^{JM}(\hat{r}, \eta_1, \eta_2) \tag{31}$$

with

$$\Phi_{Rjl}^{JM}(\hat{r}, \eta_1, \eta_2) = \sum_{m_R m_l m_s} C_{m_l m_R M}^{lRJ} C_{m_l m_s m}^{lsj} \Phi_1^{Rm_R}(\eta_1) \{\phi_2(\eta_2) i^l Y_{lm_l}(\hat{r}) \sigma_{sm_s}\} \tag{32}$$

Let us denote the matrix elements of the interaction potential by :

$$< R, l; J \mid V \mid R^{'}, l^{'}; J >= \delta_{RR'} \delta_{ll'} V_0(r) + W_2(R, l, R^{'}, l^{'}; J) V_2(r), \tag{33}$$

where

$$W_2(R, l, R^{'}, l^{'}; J) \equiv < \Phi_{Rjl}^{JM} \mid (4\pi/5)^{1/2} Y_{2m}(\hat{r}) Y_{2m}(\eta_1) \mid \Phi_{R' j' l'}^{JM} >$$

Substituting (31) into the homogeneous and inhomogeneous equations [6]

$$[H - E] \mid \chi_E^0 >= 0. \tag{34}$$

$$[H - E] \mid \chi_E^\nu >=< \Psi_\nu \mid H \mid \chi_E^\nu >\mid \Psi_\nu > \tag{35}$$

and using (34) we obtain the systems of coupled differential equations (9) and (10) for the radial wave functions $u_{Rjl}^{J(o,\nu)}$. In (9) and (10) we have dropped the subscripts $J$and also $j$since for the spinless particle $s = 0$and $j = l$.