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Some considerations about mixing, oscillation phenomena and CP violation of different mass eigenstates

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Abstract. In the last twenty years many experimental and theoretical efforts have been made to measure and verify the predictions of the Standard Model, as well as to put in evidence small deviations from these values.
In the present paper, we discuss some aspects of possible regularities in the formalism of mixing, oscillation and CP violation phenomena of the mass eigenstates in particle physics, and eventual consequences of hidden proprieties of the systems.
In the literature, different parameterising representations of lepton flavour mixing are used for three generations of leptons and quarks. Although these are mathematically equivalent, only one of them are likely to describe the underlying physics in a more transparent way, and are particularly convenient in the analysis of experimental data or is able to establish a concordance with these ones.
Flavour permutational symmetry of the mixing matrix, "maximal democracy" and maximal CP violation give a simple and efficient way to understand the essential characteristics of the phenomena, with a minimum number of parameters. The deviations of the experimental data from these predicted values represent a simple clue to study phenomenologically the breaking of symmetries.
The interplay of gravitation and linear superposition of different mass eigenstates, and their consequences on the oscillation clocks, phases and the physical observability are also briefly discussed.

Introduction

An understanding of flavour mixing and CP violation, observed in the weak interactions, remains one of the major challenges in particle physics. At the present time, one hopes that a complete solution to the fermions mass and flavour-mixing problem is possible to be achieved. In any quantum mechanical system the mixing pattern of the states will influence the pattern of the mass eigenstates, and vice-versa. One possible way to make these links more transparent is to look for specific symmetry limits.
In the present paper, we discuss at the phenomenological level some aspects of possible regularities of the formalism of mixing and oscillation phenomena of the mass eigenstates in particle physics, and eventual consequences of hidden proprieties of the systems. Flavour permutational symmetry of the mixing matrix, "maximal democracy" and maximal CP violation give a simple and efficient way to understand the essential characteristics of the phenomena, with a minimum number of parameters. The deviations of the experimental data of these predicted values represent a simple clue to study phenomenologically the breaking of symmetries.
In the last twenty years, many experimental and theoretical efforts have been made to measure and verify the predictions of the Standard Model, as well as to put in evidence small deviations from these values.
The understanding of the neutrino phenomena and CP violation in different hadronic and leptonic systems can be a very useful clue for other phenomena, as well as, e.g. lepton number violation.
The interplay of gravitation and linear superposition of different mass eigenstates, and their consequences on the oscillation clocks, phases and physical observables are also briefly discussed.

1. Descriptions of quark or lepton mixing and CP violation

1.1 Mass hierarchy, mixing and quark/lepton mixing matrix parameterisation

The Standard Model starts out by postulating chiral symmetries for the interactions of the elementary fermions and in accord with this hypothesis the elementary fermions have a zero mass. Their finite masses are attributed to symmetry breaking by the so-called Higgs mechanism. An important aspect is that this mechanism does not predict the masses, but can be accommodated with particle world. At the present time "the periodic system of elementary fermions" can be arranged in three generations. In corresponding positions in each generation, the quarks have the charge +2/3 and -1/3, as u_i and d_i, respectively, and to the charged leptons and their associated neutrinos as e_i and ν_i, respectively (charged leptons have the charge -1 and their associated neutrinos have zero charge). In this symmetry arrangement, hierarchical masses increase from one generation to the next one. In Table 1, the three generations of elementary fermions are indicated. The values of the masses are taken from RPP [1]. For neutrino masses, there are only model limits [2]. All the masses are in expressed in MeV.
The four known elementary interactions (strong, electromagnetic, weak and gravitational) are exhibited by the different types of elementary fermions. The number of possible interactions decreases from quarks to charged leptons and to neutrinos. The approximate relative strengths of the interactions are found to be independent of the generation number.

Table 1. The three generations of elementary fermions. All masses are in MeV except these for neutrinos.

		Mass [MeV]		Mass [MeV]
$i=3$	t	$(1.74\pm.05)\times10^5$	τ	1.777×10^5
	b	$(4\div4.3)\times10^3$	ν_τ	$\approx10^{-2}\ [eV]$
$i=2$	c	$(1.15\div1.35)\times10^3$	μ	105.7
	s	$(75\div170)$	ν_μ	$\approx10^{-2}\ [eV]$
$i=1$	u	$(1.\div5)$	e	0.511
	d	$(3\div9)$	ν_e	$\approx10^{-2}\ [eV]$

The mixing of either u_i or d_i mass quark eigenstates through their weak interactions can be represented by a Cabbibo-Kobayashi-Maskawa (CKM) unitary matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1}$$

with the following measured ranges for the matrix elements:

$$\begin{pmatrix} .9742 \div .9757 & .219 \div .226 & .002 \div .005 \\ .210 \div .225 & .9734 \div .9749 & .037 \div .043 \\ .004 \div .014 & .035 \div .043 & .9990 \div .9993 \end{pmatrix} \quad (2)$$

A similar mixing matrix, the Maki-Nakagawa-Sakata, exists for charged and neutral leptons.

$$\begin{pmatrix} V_{1e} & V_{1m} & V_{1t} \\ V_{2e} & V_{2m} & V_{2t} \\ V_{3e} & V_{3m} & V_{3t} \end{pmatrix} \quad (3)$$

Neutrinos require a special treatment because the mechanism of generates the masses is different.
M. Goldhaber [3] has established some empirical regularities resulting from CKM matrix useful in the understanding of some qualitative properties of elementary fermions and of their interactions. In its view, every elementary fermion of different generations is associated with identical elementary interactions and a correlation between the mass of a fermions and the relative strength of its dominant interaction could be established too. For each elementary fermion, besides its dominant interaction, it posses all the weaker ones. In the CKM matrix, the experimental values of the elements decrease with the increase of the difference between the generations, from $0 \le |i-j| \le 2$.

Recently, J. Bordes et. al. [4] provided a possible explanation of the intricate and otherwise mysterious mass and mixing patterns of quarks and leptons. This model also offers an explanation for the existence of exactly 3 fermion generations and a suggestion of how the symmetry is broken was done.

1.2 Possible parameterisations of mixing matrix (CKM, MNS matrix)

In general, the flavour mixing among N different lepton or quark families is described by a unitary matrix with a dimension $N \times N$, whose number of independent parameters depends on the nature of particles. In the case of neutrinos, if neutrinos are Dirac particles, the unitary matrix V can be parameterised in terms of $N(N-1)/2$ rotation angles and $(N-1)(N-2)/2$ phase angles. If the neutrinos are Majorana particles, however, a full parameterisation requires $N(N-1)/2$ rotation angles and the same number of phase angles. Formally, the flavour mixing of charged leptons and Dirac neutrinos is completely analogous to that of quarks, for which a number of different parameterisations have been proposed in the literature, but there are not arguments to suppose identical forms for these. Although different representations of lepton flavour mixing are mathematically equivalent, one of them is very likely to describe the underlying physics of lepton mass generation and CP violation in a more transparent way, or is particularly convenient in the analyses of experimental data.
If we consider the lepton flavour mixing in the form of CKM matrix, the strength of CP or T violation in normal neutrino oscillations (no matter whether neutrinos are Dirac or Majorana particles) depends only upon a universal parameters V_{ai}, defined as an invariant operator J through the following equation:

$$\mathrm{Im}\left(V_{ai} V_{bj} V_{aj}^{*} V_{bi}^{*}\right) = J \sum_{g,k} e_{abg} e_{ijk} , \quad (4)$$

where the Greek and Latin subscripts run over (e, m, t) and over $(1,2,3)$ respectively.

In the Standard Model there is a single source of CP violation, which is a CP violating phase d , the consequence of which is that all CP observables are strongly correlated. From the point of view of the fundamental theory of quarks and leptons this situation is not satisfactory.
The CKM matrix of quark weak couplings can be written, in the Wolfenstein parameterisation, as

$$V=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-id_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{id_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{id_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{id_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{id_{13}} & c_{23}c_{13} \end{pmatrix}. \qquad (5)$$

The CP and T violation effects are measured by

$$J=s_{12}c_{12}s_{23}c_{23}s_{31}c_{31}^2 sin d\ . \qquad (6)$$

An equivalent form is:

$$V\cong\begin{pmatrix} 1-l^2/2 & l & Al^3(r-ih) \\ -l & 1-l^2/2 & Al^2 \\ Al^3(r-ih) & -Al^2 & 1 \end{pmatrix}+O\left(l^4\right)+X \qquad (7)$$

The parameterisation is a convenient way to make the unitarity of the matrix explicit, up to higher order corrections in powers of $l\equiv V_{us}$. The quantity l is essentially the sine of the Cabibbo angle. It is a small number, of the order 0.2. The other independent magnitude parameters A and r^2+h^2 are known to be roughly of the order of unity. There is no theory behind, where powers of l enter in each term. This parameterisation simply summarises the observations in a simple way [5]. The $O\left(l^4\right)$ term represents a correction to the actual parameterisation.
The lepton flavour-mixing matrix can be parameterised in close analogy with that for quark flavour mixing. In fact, for $N=3$ particles, on can find nine distinct ways to describe mixing for 3×3 matrix.
Note that, for Majorana particles, there appear three CP violating phases [6], the Dirac phase d and the Majorana phases b, r , represented as a phase matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ir} \end{pmatrix} \qquad (8)$$

2. Schemes for neutrinos mixings

2.1 Some useful experimental results

In this paper the expression for the probability of $n_a \to n_b$ neutrino transition in vacuum will be considered in the standard form, see for example references [7] and [8].

Reactor experiments are realised for the L/E in the range $2 \div 50\, m/MeV$. Accelerator experiments have explored the region $L/E \leq 1 km/GeV$. The sources of cosmic neutrinos are atmospheric neutrinos. In these experiments, the neutrino covers a wide range $2 \leq L/E \leq 2 \times 10^4\, km/GeV$ and the solar neutrino data cover the range $10^{10} \leq L/E \leq 10^{12}\, m/MeV$.

Some useful experimental data are reviewed. From the CHOOZ experiment, the data agree with a sine value $sin^2 \boldsymbol{J}_{13} < 0.03\ (90\%CL)$. Assuming the LMA solution for solar neutrinos combined with the KamLAND result [9], $\Delta m^2_{Solar} \cong 7.32 \times 10^{-5} eV^2$ and $\Delta m^2_{Atm.} \cong 2.5 \times 10^{-3} eV^2$ respectively and $|U_{e3}| \leq 5 \times 10^{-2}\ \ (99.7\%CL)$. Also, the mixing angle is $\sin^2 2\boldsymbol{J}_{atm} > 0.84$.
From a two neutrino analysis of the data, in the LSND experiment, the best-fit value of the oscillation parameters are $\Delta m^2_{LSND} \cong 1.2\ eV^2$ and $sin^2_{LSND} 2\boldsymbol{J} \cong 0.003$, but a confirmation is required. Giunti did a more complete compilation in refs. [10] and [11].

2.2 Threefold maximal mixing

Vacuum neutrino oscillations are conveniently discussed in a weak basis, which diagonalises the mass matrix of the charged leptons. The very simply hypothesis is the threefold maximal mixing [12], [13], [14]. The mixing matrix can be written in the form:

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} w_1 & w_1 & w_1 \\ w_1 & w_2 & w_3 \\ w_1 & w_3 & w_2 \end{pmatrix} \quad (9)$$

where w_i with $i = 1,2,3$ are the complex cubic roots of unity. The mixing is maximal in that all the elements of the mixing matrix have equal modulus $|U_{nl}| = 1/\sqrt{3}$. In this case, Jarlskog parameter is extremal $|J| = 1/(6\sqrt{3})$ such that (vacuum) CP and T violating asymmetries are maximised. In this mixing scheme the probabilities for $l \to l$ mixing are:

$$P(l \to l)\big|_{l=e,m,t} = |p|^2 = \frac{1}{3} + \frac{2}{9}(c_{12} + c_{23} + c_{31}); \quad (10a)$$

For the oscillations of the type: $e \to m \to t \to e$ or $\bar{e} \to \bar{t} \to \bar{m} \to \bar{e}$ the transition probabilities are:

$$P(l \to l') = |q|^2 = \frac{1}{3} - \frac{1}{9}(c_{12} + c_{23} + c_{31}) + \frac{1}{3\sqrt{3}}(s_{12} + s_{23} + s_{31}) \quad (10b)$$

and for the processes: $e \to t \to m \to e$ and $\bar{e} \to \bar{m} \to \bar{t} \to \bar{e}$ the transitions probabilities are:

$$P(l \to l'') = |r|^2 = \frac{1}{3} - \frac{1}{9}(c_{12} + c_{23} + c_{31}) - \frac{1}{3\sqrt{3}}(s_{12} + s_{23} + s_{31}) \text{ respectively.} \quad (10c)$$

In this mixing scheme, the various survival probabilities $P(l \rightarrow l)\big|_{l=e,m,t}$, as measured in disappearance experiments, are identical, so that there is a universal function of L/E , independent of generation. This prediction is in accord with the solar data for the probability $P(e \rightarrow e) \cong 5/9$, independent of energy, for values of the energy of the order $E > 5 MeV$ for SAGE, GALLEX, HOMESTAKE and Super-K experiments, if $\Delta m'^2 = 1.0 \times 10^{-3} eV^2$. This result is supported by Ahluwalia's analysis starting from the experimental results [15].

2.3 Bi-maximal neutrino mixing and possible invariance

Bi-maximal mixing could be seen as just the minimal deformation of tri-maximal mixing obtained enforcing a zero in the top right-hand $(e3)$ corner of the tri-maximal mixing matrix. This fact is justified by the atmospheric data. The symmetry between all three generations is sacrificed in this bi-maximal scheme [16], [17] in accord with the matrix:

$$\left(\left|U_{ln}\right|^2\right) = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} \qquad (11)$$

but clearly $m \leftrightarrow t$ symmetry, or equivalently $n_1 \leftrightarrow n_2$, do survive.
Xing [18] proposed a simple extension of the current bi-maximal neutrino-mixing pattern so that large CP violation in the lepton sector could be accommodated and much flexibility in accounting for the solar neutrino problem is allowed, supposing that the neutrinos are Dirac particles:

$$V = \begin{pmatrix} \frac{\cos J}{\sqrt{2}} & \frac{\cos J}{\sqrt{2}} & -i\sin J \\ -\frac{1+i\sin J}{2} & \frac{1-i\sin J}{2} & \frac{\cos J}{\sqrt{2}} \\ \frac{1-i\sin J}{2} & -\frac{1+i\sin J}{2} & \frac{\cos J}{\sqrt{2}} \end{pmatrix} \qquad (12)$$

This angle measures a slight coupling between solar and atmospheric oscillations. Thus, the CP violation turns out to be:

$$J = \frac{\sin J \times \cos^2 J}{4} \qquad (13)$$

where if we consider $\sin^2 2J_{Sun} \cong \sin^2 2J_{Atm} \cong 1$, thus $J \le \frac{1}{8\sqrt{2}} \cong 0.088 < J_{\max} = \frac{1}{6\sqrt{3}} \cong 0.096$.

The CP violation signal could be from the probability asymmetry between $n_e \Leftrightarrow n_m$ from solar neutrino.
Scott [14] suggests an 'optimised' bi-maximal mixing matrix in the aim to explain latest Super-K solar data:

$$\left(\left|U_{ln}\right|^2\right) = \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix} \tag{14}$$

with $\Delta m'^2 = 5.6 \times 10^{-5}\, eV^2$.

Recently, in a very short note [19, Tangoç Rador suggest that for every mass matrix there exist a basis, which may be not the weak eigenbasis, in which this matrix will be with near norm democratic.

2.4 Other possibilities

In the chiral limit, $m_u \to 0$ and $m_d \to 0$, the flavour mixing angles implying the up and down quarks are also zero and the flavour mixing matrix effectively takes the particular form:

$$V \cong \begin{pmatrix} \cos J & \sin J \\ -\sin J & \cos J \end{pmatrix}, \tag{15}$$

One angle is necessary. This approximation has not effect on CP symmetry in the $B - \overline{B}$ system for example, but it could affect the magnitude of mixing angle. The other extreme case is the heavy quark limit, with $m_b \to \infty$ and $m_t \to \infty$. In this limit the (t,b) system is decoupled from (c,s) and (u,d) systems, and the flavour mixing can be described for a case of two families, and the mixing matrix is:

$$V \cong \begin{pmatrix} s_u s_d + c_u c_d e^{-ij} & s_u c_d - c_u s_d e^{-ij} \\ c_u s_d - s_u c_d e^{-ij} & c_u c_d + s_u s_d e^{-ij} \end{pmatrix} \tag{16}$$

with the values of angles modified because they are obtained in this limit.
At the level of quark mass, note that CP symmetry would be conserved if two quarks with the same charge would have degenerate mass eigenvalues. At the level of the flavour-mixing matrix, CP symmetry is violated if V contains a nontrivial complex phase, which cannot be removed through the redefinition of quark-field phases. The possibility of maximal Jarlskog invariant J and maximal CP violating phase is investigated by Rodriguez-Jauregui [20]. In accord with their study, the Jarlskog invariant takes its extremum value when the first derivative of J with respect to the CP violating phase d vanishes:

$$\left.\frac{\partial J}{\partial d}\right|_{d^*} = 0 \tag{17}$$

Three types of solutions are possible: the values 90° and 270° correspond to maximal invariant J and maximal CP violating phase. The solution 45° and 135° corresponds to maximal invariant J and non-maximal CP violating phase, and for the solution 0° and 180° J has an inflection point, CP is an exact symmetry and Jarlskog's invariant is zero.

For neutrinos, an important consequence of the smallness of $\left|U_{e3}\right| \le 5 \times 10^{-2}$ $(99.7\% CL)$ is that the solar and atmospheric neutrino oscillations are decoupled and the experimental results could be analysed in terms of two states. Thus, for atmospheric transitions (between the $n_m \leftrightarrow n_t$ states), the mixing angles are given by:

$$\sin^2 \boldsymbol{J}_{atmospheric} = \frac{\left|U_{m3}\right|^2}{1-\left|U_{e3}\right|^2} \qquad \cos^2 \boldsymbol{J}_{atmospheric} = \frac{\left|U_{t3}\right|^2}{1-\left|U_{e3}\right|^2} \tag{18}$$

and for solar neutrino oscillations (between the $\boldsymbol{n}_e \leftrightarrow \boldsymbol{n}_m$ states):

$$\sin^2 \boldsymbol{J}_{solar} = \frac{\left|U_{e2}\right|^2}{1-\left|U_{e3}\right|^2} \qquad \cos^2 \boldsymbol{J}_{solar} = \frac{\left|U_{e1}\right|^2}{1-\left|U_{e3}\right|^2} \tag{19}$$

Using the available atmospheric and solar data from Super-K, LMA, LOW and CHOOH experiments, C. Giunti and M. Tanimoto [17] and J.H. Field [21] get the allowed intervals for the absolute values of the mixing matrix:

$$|U| \cong \begin{pmatrix} .67 \div .91 & .41 \div .73 & .00 \div .22 \\ .20 \div .70 & .21 \div .81 & .51 \div .87 \\ .05 \div .66 & .37 \div .83 & .49 \div .85 \end{pmatrix} \tag{20}$$

Because of the large ambiguities in the evaluated mixing matrix values, the rules observed by Goldhaber in relation with CKM matrix [3] can be only partially verified.

Beyond the standard source of CP violation, from the mismatch between the mass quark and weak interaction and possibly lepton eigenstates, if the noncommutative Standard Model (ncSM) is considered [22], [23], there is in additional source of CP violation.

Noncommutative space-time is a deformation of ordinary space-time in which the space-time coordinates (x_m), representable by the Hermitian operators $\hat{x}_m$ do not commute: $\left[\hat{x}_m, \hat{x}_n\right] = i\,\boldsymbol{J}_{mn}$. Here $\boldsymbol{J}_{mn}$ is the deformation parameter; and the ordinary space-time is obtained in the $\boldsymbol{J}_{mn} \to 0$ limit. This parameter is the smallest patch of area in the in the $\boldsymbol{mn}$-plane one could deem observable. The average magnitude of this parameter is $\boldsymbol{J}$, and $1/\sqrt{\boldsymbol{J}}$ corresponds to the energy threshold beyond which the space-time is distorted. If the action is considered for a noncommutative geometry as in the cited papers, thus, under the discrete symmetries P and C one obtains the following relations: for parity transformation, $x_i \to -x_i$, $A_0 \to A_0$, $A_i \to -A_i$, $\Psi(x) \to \boldsymbol{g}^0 \Psi(x)$, $\boldsymbol{J}_{mn} \to \boldsymbol{J}_{mn}$ leaves the action invariant, and under charge conjugation $\partial_m \to \partial_m$, $A_m \to -A_m$, $\boldsymbol{J}_{mn} \to -\boldsymbol{J}_{mn}$

Therefore, under C and P combined $\boldsymbol{J} \to -\boldsymbol{J}$ can actually explain the origin of CP violation.

The overall effect of noncommutativity is to introduce momentum-dependent phases into the CKM matrix. In accord with [22, 23] the CKM matrix must be modified introducing an additional matrix term X (see equation 7):

$$X = \begin{pmatrix} ix_{ud} & ix_{us} & iA\,\boldsymbol{l}^3 x_{ub} \\ -ix_{cd} & ix_{cs} & iA\,\boldsymbol{l}^2 x_{cb} \\ iA\,\boldsymbol{l}^3\,\boldsymbol{r}x_{td} & iA\,\boldsymbol{l}^2 x_{ts} & ix_{tb} \end{pmatrix} \tag{21}$$

where $x_{ab} \equiv p_a^m \boldsymbol{q}_{mn}\, p_b^n$ for quarks a, b.

Thus the Jarlskog invariant is $J \approx A^2 \boldsymbol{l}^6 \boldsymbol{h} - \boldsymbol{l}^2 p^2 \boldsymbol{J}$, p being the typical momentum flow in the process. The first term result comes the standard parameterisation and it is of the order 2.53×10^{-5} if the following values for parameters $A \cong 0.81$, $\boldsymbol{l} \cong 0.22$, $\boldsymbol{h} \cong 0.34$ [24] are considered.
In the maximal limit for CP violation, $(p^2 \boldsymbol{J})$ is of order 1.99. Hinchliffe and co-workers calculated whether the $\boldsymbol{J}$ dependent phases are observable in K or B physics; the conclusion is that if threshold energy is approximately $1\ TeV$ then the observables of K physics are sensitive to noncommutative geometry, however for B physics they are not.

3. The effects of gravitational interaction

The classical effects of gravitation on a single mass eigenstate are usually considered in terms of a force $\vec{F}$, whereas the quantum-mechanical ones are determined by the gravitational interaction energy [25]. The interaction energy for a non-relativistic particle of mass m, in the weak-field limit of Einstein's theory, coincides with the Newton theory. Thus, for a non-rotating object of mass M, the field is $\Phi = -GM/r$ and the interaction energy is $U = m\Phi$, while $\vec{F} = -m\nabla\Phi$. Assuming the mass eigenstate to be a relativistic, one is led to the expression:

$$U = \int_{\infty}^{r} F dr' = -\frac{GME}{rc^2} = -\frac{E}{c^2}\Phi . \qquad (22)$$

In the current situation there is a linear superposition of different eigenstates with different masses, in the probability relations, their relative phases turn out to be modified through the gravitation by the mass-dependent factors of the form $\exp\left(-im_i\Phi t/\hbar\right)$. For the case considered, $\boldsymbol{j}^G = \Phi \boldsymbol{j}$, where $\boldsymbol{j}$ and $\boldsymbol{j}^G$ stand for the time oscillatory kinematical and the gravitational phase respectively. If a weak eigenstate with energy E and mass m_i is produced in the coordinate system $(\vec{r}_P, t_P)$ and detected at the coordinate $(\vec{r}_D, t_D)$ in the eigenstate mass m_j, the kinematical phase difference will be

$$\boldsymbol{j}_{ji} = \frac{c^3}{4\hbar}\frac{|r_D - r_P|\Delta m_{ji}^2}{E} = \frac{c^3}{4\hbar}\frac{L\Delta m_{ji}^2}{E} . \qquad (23a)$$

The gravitationally induced phase difference is:

$$\boldsymbol{j}_{ji}^G = \frac{GMc}{4\hbar}\int_{r_P}^{r_D}\frac{dl}{r}\frac{\Delta m_{ji}^2}{E} = -\frac{\boldsymbol{j}_{ji}}{L}\int_{r_P}^{r_D}\frac{dl}{r}\frac{GM}{c^2} = -\langle\Phi\rangle \boldsymbol{j}_{ji} . \qquad (23b)$$

Thus the Einstein's theory of gravitation requires clock red shifts.
An important question is if this contribution is relevant for the phenomena. First, it is evident that the effect is proportional with the source of gravity.
In a curved space-time, in the approximation of weak field, for radial propagation, the gravitational contribution ([26], [27]) is:

$$\langle\Phi\rangle = 1 - GM\left(\frac{1}{L^{PD}}\ln\frac{r_D}{r_P} - \frac{1}{r_D}\right) \qquad (24)$$

where r_P, r_D are the coordinates of the point that the neutrinos are produced and detected respectively, and L^{PD} is the distance between the source and the detector. Some evaluations are useful. The relevant ratio at the Earth surface is approximately $\left.\frac{\boldsymbol{j}^{G}_{ji}}{\boldsymbol{j}_{ji}}\right|_{Earth} \approx 10^{-9}$, for the Sun it is $\left.\frac{\boldsymbol{j}^{G}_{ji}}{\boldsymbol{j}_{ji}}\right|_{Sun} \approx 10^{-6}$, and respectively $\left.\frac{\boldsymbol{j}^{G}_{ji}}{\boldsymbol{j}_{ji}}\right|_{Supergalactic\ cluster} \approx 3\times10^{-5}$ for a galactic cluster, results which justify the weak approximation and permit to evaluate the number of interactions as well as to obtain a jump between $CP = +1 \Leftrightarrow CP = -1$ states (of order 10^9, 10^6 and 10^5 respectively).

Secondly, the effect of gravitationally induced phases on the oscillation probabilities is influenced also by the ratio $\frac{L}{l_{ji}}$, where $l_{ji} = \frac{4pE}{\Delta m^2_{ji}}$ is the length of oscillation for the considered eigenstates. Due to gravitational interaction

$$l_{ji} \to l^{G}_{ji} = l_{ji} - GM\left[\frac{1}{r_D}\ln\left(1 - \frac{4pE}{\Delta m^2}\right) + \frac{4pE}{\Delta m^2 r_D}\right]. \qquad (25)$$

but a confirmation is necessary.

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