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AN APPROXIMATED DELTA WAVELET

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Abstract: The wavelet theory is a powerful method for processing different signals. In this report a new wavelet was obtained. This approximated delta wavelet is based on an approximation of a delta window. From the admissibility condition in the case of wavelet regular enough we have: i) the wavelet Fourier transform in origin equal to zero and ii) the wavelet has zero mean. This transform is a more flexible one because is dependent on three parameters and can cover more combinations of discrete and continuous parameter discretisations (hybrid cases).
Keywords: Signal processing, An approximated delta wavelet

INTRODUCTION

In processing signals, a part the well-known Fourier methods, a more powerful instrument is offered by the wavelet theory [1,2]. The ortogonal base on a compact support gives the advantage of wavelets. The disavantage is implied by the necessity to pass to a space augmented by one.

METHOD

The wavelet can be constructed in many different ways. The approximated delta Window [3a, 3b] is given by: $f(e;t) = \frac{e}{e^2 + t^2}\frac{1}{p}$ (1a) Also we have : $f(e;t) = \frac{1}{e\sqrt{p}}\exp\left[-\left(\frac{t}{e}\right)^2\right]$ (1b) where for e tending to infinity the limit is the Dirac function $d(t)$. In figure 1a the approximated delta window $g(e,d;t) = \frac{e}{e^2 + (t-d)^2}\frac{1}{p}$ (2a) was given, where d is the window length. The expression (2a) is represented for different values of e,d: $(e = 0.1; e = 0.3; d = 0)$.

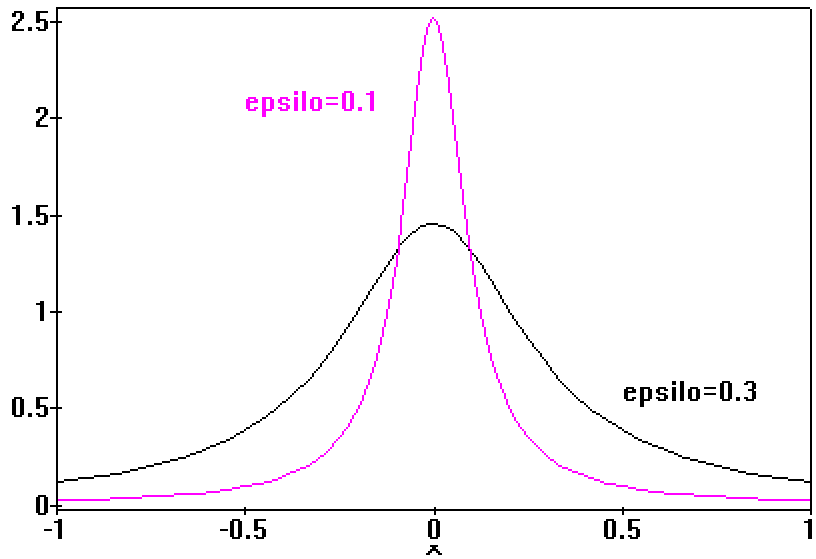
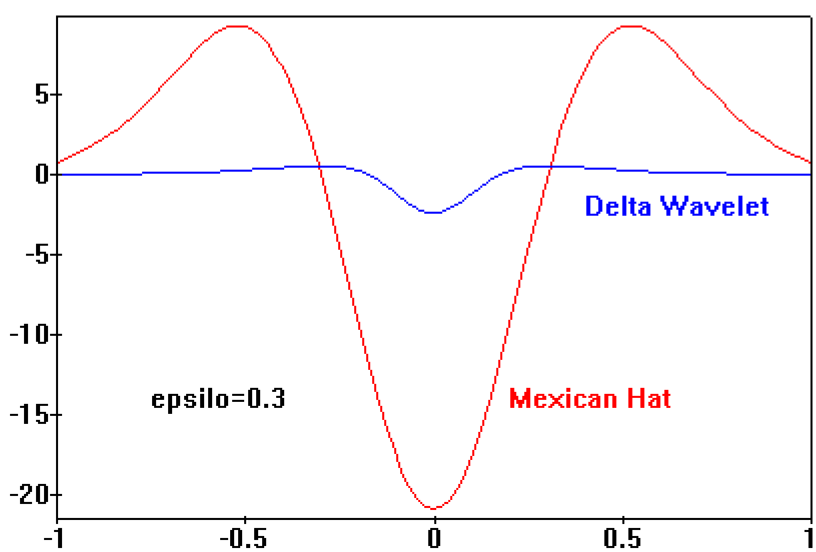


Fig.1a Normalized Delta Windows

We construct the mother wavelet which is the second derivative of expression of type (2a) : $h\left(\boldsymbol{e},0;t\right)=\dfrac{d^2 g\left(\boldsymbol{e},d;t\right)}{dt^2}$ (2b) was obtained, where d is the window length.

From the admissibility condition [4] in case of wavelet regular enough we have: i) the Fourier transform $\hat{h}\left(\boldsymbol{e},0;t\right)=0$ and ii) the mother wavelet : $\int_{-\infty}^{\infty} h\left(\boldsymbol{e},0;t\right)dt=0$ (3) . In fig. 1b the wavelets Mexican hat and deltas $\left(\boldsymbol{e}=0.2, d=0.0\right)$ were shown.



In Fig.1b Delta and Mexican hat and Wavelets

The set of daughter wavelets are generated from the mother wavelet by shift operations : $h\,(e,d;t)=\frac{1}{\sqrt{F}}h(e,0;t)$ (4a) ,where d is the shift.The normalisation factor is given by : $F=\int_{-\infty}^{\infty}\left|h(e,0;t)\right|^{2}dt$ (4b). The (1-D) wavelet transform of the f(t) was defined as :

$$W(e,d)=\int_{-\infty}^{\infty}f(t)\,h^{*}(e,d;t)\,dt \qquad (5)$$

In Fig.2a the mother wavelets with different parameters $(e=0.1, e=0.3, d=0.0)$ were represented.

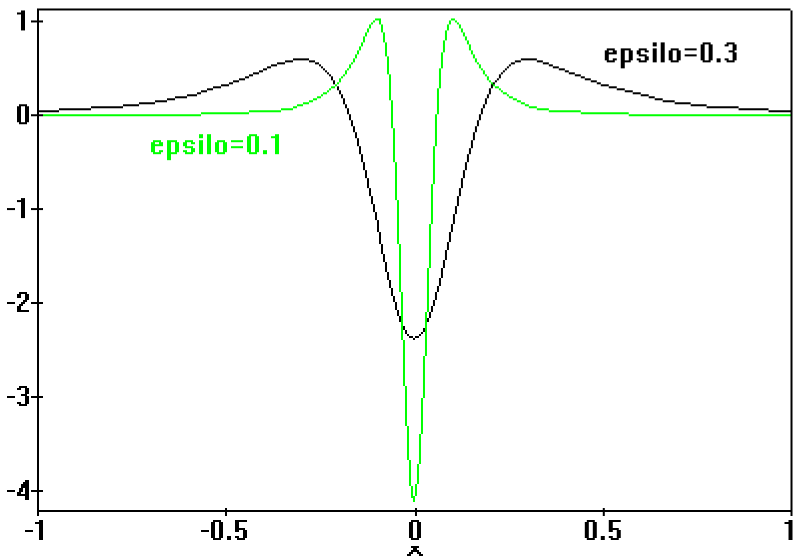


Fig.2a Normalized Delta Wavelets

The wavelet transform is a correlation operation between the signal f(t) and the shifted and scaled mother wavelet $h(e,d;t)$.

CONCLUSIONS

The new wavelet $h\,(e,d;t)$ obtained depends of three parameters (e,d). For the same number of parameters we obtained less operations in delta wavelet evaluation than for mexican hat one. So the absence of exponential evaluation for this wavelet reduce the.computing time. This type of wavelet is more versatil one and cotains the discrete and continuous parameter discretisations (hybrid cases).

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