Romanian Reports in Physics, Volume 56, Number 4, P.623-636, 2004 623

Laser modified electron bremsstrahlung revisited

M. Dondera and Viorica Florescu
Department of Physics, University of Bucharest

Received July 22, 2004

Abstract

A brief review of nonrelativistic calculations for the bremsstrahlung spectrum in the presence of a strong external monochromatic electromagnetic field is followed by an analysis of some pecularities of two approximate equations corresponding one to fast incident electrons (Karapetian and Fedorov 1978) and the other to low laser frequency regime (Zhou and Rosenberg 1993). Our numerical evaluations of the bremsstrahlung spectrum, based on these equations are reported as graphs.

Key words: bremsstrahlung, electron, laser, Coulomb field

1 Introduction

The atomic processes in the presence of a laser field are classified in two categories: laser assisted and laser induced processes. In the first case, a basic process taking place in the absence of the laser is modified by its presence, sometimes drastically. For the electron - atom (ion) system the processes from this category considered in the literature include scattering (elastic and inelastic), radiative electron capture and spontaneous emission of radiation. The laser-induced processes take place only in the presence of the external field. The most studied processes are the multiphoton ionization and the harmonic generation. There are experimental, as well as theoretical studies, for all these processes [1]. Roughly, an external field has more influence on free states than on bound states. Perturbation theory ceases to be an useful tool in the intense fields regime.

The transitions taking place during electron scattering, induced or spontaneous, are also called *free-free transitions* because the initial and final states of the electron are both in the continuum.

We refer here to *the laser modified electron bremsstrahlung*, a laser-assisted process. The process in the absence of the field is the ordinary bremsstrahlung, i.e., the *spontaneous emission of a photon* in the presence of an atomic target (atom, ion, nucleus). Essential features of the electron bremsstrahlung on ionic targets can be understood using the Coulomb potential [2] as a model for the target. The presence of an external electromagnetic field modifies the emission of radiation during scattering. Attention was given in the literature to *laser induced bremsstrahlung* and *inverse bremsstrahlung*, processes in which the electron is forced to emit and, respectively, absorb laser photons. A problem of practical interest for plasma physics is to establish which of induced emission or absorption is predominant. The spontaneous emission takes place simultaneously with induced transitions and its features are expected to be modified by the external intense electromagnetic field. For this case only theoretical investigations can be found in the literature. For a brief review of the literature up to 1993, we mention §4.5 of the Physics Report review of Ehlotzky *et al.* [3]. The situation has not changed much since.

In the simplest description, adopted here, the atomic target is replaced by a potential field. Even in this case, the majority of the existing studies on laser assisted spontaneous bremsstrahlung contain qualitative conclusions only. In an effort to explore the available information, we have analyzed two of the existing equations, based on different assumptions:

i) the Born approximation calculation of Karapetian and Fedorov [4]

ii) the low laser frequency approximation of Zhou and Rosenberg [5].

Both calculations are nonrelativistic and refer to a monochromatic electromagnetic field.

We first refer briefly in Sect. 2 to the ordinary bremsstrahlung. We do this because for laser modified bremsstrahlung we meet the same type of cross sections as for ordinary bremsstrahlung, and more than that, in the approximate equation of Zhou and Rosenberg in Sect. 6 we meet the ordinary bremsstrahlung cross sections with modified parameters. In Sect. 3 we give a qualitative description of spontaneous emission in the presence of a laser. A brief mention to the literature is the objective of Sect. 4. The two approximations we are interested in are presented in Sects. 5 and 6. A feature of the bremsstrahlung spectrum predicted by Karapetian and Fedorov is a resonant behaviour in very intense fields. The transition from a flat spectrum at low intensities to an oscillatory one in an intense field is the subject of our discussion in Sect. 5. Then, in Sect. 6, we analyze one of the two expressions for the spectrum derived by Rosenberg and Zhou. We extract its consequences for the extension of the spectrum beyond the ordinary bremsstrahlung tip and discuss it in terms of the maximum number of emitted and absorbed laser photons. For the Coulomb field we present as graphs the behaviour of spectra with increasing external field intensity. Finally, in Sect. 7 we compare the predictions of the two approaches and focus on their range of validity.

2 General equations for ordinary bremsstrahlung

In ordinary bremsstrahlung [2], during a collision the electron losses part of its energy by the emission of one photon of angular frequency denoted by $\omega_{\rm X}$. The energy is conserved, so the final electron energy is

$$T_f = T_i - \hbar\,\omega_{\rm X}\,, \tag{1}$$

where T_i is the incident electron energy. A momentum $\mathbf{q}$ is transfered to the atomic field

$$\mathbf{q} = \mathbf{p_i} - \mathbf{p_f} - \hbar\,\boldsymbol{\kappa}_{\rm X}\,, \tag{2}$$

where $\mathbf{p_i}$ and $\mathbf{p_f}$ denote, respectively, the initial and final electron momentum and $\hbar\,\boldsymbol{\kappa}_{\rm X}$ the photon momentum. As in any spontaneous emission, all directions and polarizations are possible for the photon. The emitted radiation spectrum extends from 0 to the tip frequency

$$\omega_{\rm tip}^{\rm spont} = T_i/\hbar\,. \tag{3}$$

We refer to the case where the photon polarization and the electron spin are not observed. In a nonrelativistic treatment the electron spin is ignored.

The simplest and most common observation detects the photon spectrum only and leads to the cross-section

$$d\sigma_{\rm X}^{\rm spont}(T_i,\omega_{\rm X}) \equiv \frac{\text{spont. emission rate in } (\omega_{\rm X},\omega_{\rm X}+d\omega_{\rm X})}{\text{incident electron flux}} = \frac{d\sigma_{\rm X}^{\rm spont}}{d\omega_{\rm X}}d\omega_{\rm X}, \tag{4}$$

which depends on the incident electron energy, and changes with the emitted frequency. Obviously, the cross section depends also on the atomic potential. The spectrum is obtained by summing over the photon polarization, integrating over its direction and integrating over the scattered electron attributes.

To a more elaborate experiment, which detects also the photon direction, it corresponds a double differential cross section, $d^2\sigma_{\rm X}^{\rm spont}/d\omega_{\rm X}\,d\Omega_{\rm X}$, where $d\Omega_{\rm X}$ is the solid angle element in which the photon is emitted. Another double differential cross section corresponds to the detection of the photon spectrum in coincidence with the electron direction. This cross section will be denoted by $d^2\sigma_{\rm e,X}^{spont}/d\omega_{\rm X}\,d\Omega_e$, with $d\Omega_e$ the solid angle element in which the electron is emitted.

3 Laser modified bremsstrahlung

In the presence of an electromagnetic field, the cross sections mentioned previously are *defined* in the same way as before, but depend on the field parameters. In this paper, the external field will be taken as monochromatic, with frequency $\omega_{\rm L}$ and maximum intensity I_L. In this case, the most intuitive and usual description of the influence of the laser on spontaneous emission will envisage it as taking place simultaneously with the emission or (and) the absorption of laser photons. The theory supports directly this interpretation, because if the interaction with the external field lasts long enough, any of the cross sections defined previously can be presented as a sum of cross sections, each term in the sum corresponding to the spontaneous emission of a photon of frequency $\omega_{\rm X}$ accompanied by a net exchange of n laser photons; $n > 0$ and $n < 0$ mean, respectively, net absorbtion and emission of $\mid n \mid$ laser photons. The situation is not easy to interpret if the laser pulse is short.

So, in the presence of the field, the cross-section $d\sigma_{\rm X}$, defined by (4), can be presented as:

$$d\sigma_{\rm X} = \sum_{n=n_0}^{\infty} d\sigma_n \, . \tag{5}$$

The minimum number of exchanged laser photons was denoted by n_0 . In the present context, we shall call $d\sigma_n$ a partial cross section. The final electron energy, different for each term, is

$$T_n = T_i - \hbar\,\omega_{\rm X} + n\,\hbar\,\omega_{\rm L} \geq 0 \tag{6}$$

Theoretically, in contrast with ordinary bremsstrahlung, the spontaneously emitted photon frequency $\omega_{\rm X}$ can be arbitrarily large, if the necessary large number of laser photons is absorbed by the electron.

For $\hbar\omega_{\rm X} \leq T_i$, a negative n is allowed only if

$$\mid n \mid \leq \frac{T_i - \hbar\omega_{\rm X}}{\hbar\,\omega_{\rm L}}\, , \qquad n < 0\, . \tag{7}$$

For $\hbar\omega_{\rm X} \geq T_i$, the number of absorbed photons should satisfy

$$n \geq \frac{\hbar\omega_{\rm X} - T_i}{\hbar\,\omega_{\rm L}} \geq 0\, . \tag{8}$$

In both cases there is not a superior margin for n. Nevertheless a decrease of the cross section $d\sigma_n$ with increasing n is expected, so practically there is an endpoint of the photon spectrum. Some of the existing approximate equations predict an end point of the spectrum. More on this will be said in Sect. 6.

Each partial cross section depends on the external field parameters; in particular $d\sigma_0$ is not the ordinary bremsstrahlung cross section, but reduces to it in the absence of the field. The other partial cross sections ($n \neq 0$) vanish in the absence of the external field.

We shall not give details on the derivation of the equations in the following sections. The most convenient procedure is a hybrid one, describing the electromagnetic field as having two components, one described classically, corresponding to the laser field, and the other one, needed to describe the spontaneous emission, quantized. The calculations considering that *only one photon is emitted spontaneously* treat the interaction with the quantized field as a perturbation.

4 Low and intermediate intensity calculations

The literature of spontaneous emission in electron scattering in the presence of an external field starts with relativistic calculations [6]. It was later realized that the nonrelativistic regime for electron energy should be more interesting for experiment [4].

At low electromagnetic field intensity one can use perturbation theory. Its limit of valability is not well established, but it was applied to intensities up to 10^9 W/cm^2. The number of laser photons implied in a free-free transition (some of them virtually) gives the needed order of the perturbation theory to be used in the calculation. In the Coulomb case the description of the induced emission or absorption of only one laser photon is possible using adequately [7] the well known Sommerfeld formula.

Spontaneous emission simultaneously with the exchange of a given number of laser photons was studied in the cases $n = \pm 1$ for electron scattering in a potential. This was done in second order perturbation theory. The calculation of Krainov and Roshchupkin [8] is valid at low energy electrons scattered by the Coulomb field. High energy results can be extracted from Karapetian and Fedorov equation [Eq. (9) below], valid for any laser intensity. A calculation at any nonrelativistic electron energy, but valid at low field intensity, is possible based on the knowledge of two-photon continuum-continuum matrix element in the Coulomb field [9, 10, 11, 12]. Nevertheless, numerical calculations have not be done, with one exception: an evaluation of the cross-section $d^3\sigma_1^{\rm PT}$, corresponding to the detection of electrons and photons in coincidence, only for the very particular geometry in which the laser linear polarization $\mathbf{s}_{\rm L}$, the emitted photon polarization $\mathbf{s}_{\rm X}$ and the incident electron momentum $\mathbf{p_i}$ are orthogonal to each other. The published results [13] correspond to low incident electron energy, arround 10 eV and external frequency $\omega_{\rm L} = 1.17$ eV (Nd:YAG laser case). A comparison was made with ordinary bremsstrahlung emitted under the same condition, $\mathbf{s_X} \cdot \mathbf{p_i} = 0$, with the conclusion that in the incident energy range investigated (10-30 eV) the Compton scattering in the presence of the Coulomb potential dominates ordinary bremsstrahlung for $\rm I_L/I_0 > 10^{-8}$ ($\rm I_0 \approx 3.5 \times 10^{16}$ W/cm^2 is usually taken as the unity of electromagnetic radiation intensity).

The extension to intermediate laser intensity (the range $10^7 - 10^{12}$ W/cm^2)

was done in a recent calculation [14] of the spontaneous bremsstrahlung accompanied by an exchange of 0, 1 and 2 laser photons. The approximation of the total cross section based only on the inclusion of the mentioned terms indicates a reduction of the spontaneous emission compared with the case in which the laser field is absent (see Fig. 5 of [14]).

5 Born approximation

In Born approximation the electromagnetic field is considered so intense that the atomic potential could be treated as a perturbation. A calculation was performed by Karapetian and Fedorov [4] in the case of the Coulomb potential, in the nonrelativistic case and in dipole approximation, for a monochromatic linearly polarized electromagnetic field. In our notations, their expression for the double differential cross section that would correspond to detecting the emitted X photon frequency and the scattered electron direction in coincidence, is

$$d^2\sigma^{\rm KF}_{\rm e\,X} = \sum_{n_0}^{\infty} d^2\sigma^{\rm KF}_n$$
$$d^2\sigma^{\rm KF}_n = \frac{8}{3\,\pi}\, Z^2\,\alpha^5\,a_0^2\,\frac{p_n}{p_i}\,\frac{m_e^2\,c^2}{q_n^2}\,J_n^2(z_n)\,d\Omega_e\,\frac{d\omega_X}{\omega_X}\,. \tag{9}$$

with

$$p_n^2 = p_i^2 + 2\,m_e\,\hbar\,\left(n\,\omega_{\rm L} - \omega_{\rm X}\right)\,, \qquad z_n = \frac{\alpha_{\mathbf{0}}\,\mathbf{s}_{\rm L}\cdot\mathbf{q_n}}{\hbar}\,, \qquad \mathbf{q_n} \equiv \mathbf{p_i} - \mathbf{p_n}\,. \tag{10}$$

The vector $\mathbf{p_n}$ is the momentum vector of the scattered electron (mass m_e) which has exchanged n photons with the laser field. The vector $\mathbf{q_n}$ is the momentum transfer from the electron to the atomic field, in dipole approximation. As usually, the fine structure constant is denoted by α, the first Bohr radius by a_0 and the velocity of light by c. The real unity vector $\mathbf{s}_{\rm L}$ describes the direction of the laser polarization. The quantity α_0 ,

$$\alpha_0 = -\frac{e\,A_0}{m_e\,\omega_{\rm L}} \tag{11}$$

coincides with the amplitude of the quiver motion of an electron in a monochromatic homogeneous electric field of maximum intensity $\mathcal{E}_0 = \omega_{\rm L}\,A_0$. The use of the Born approximation implies $\alpha\,Z\,c/v \ll 1$, with Z the charge of the nucleus and v the electron velocity. The condition has to be fulfilled by both the incident and scattered electron velocities.

For very low laser frequency the momentum p_n is replaced by p_0. This implies

$$d^2\sigma_n^{\rm KF,soft} = d^2\sigma^{\rm spont}_{\rm e X}\,J_n^2(z_0)\,, \qquad d\sigma_{\rm X}^{\rm KF,soft} = d\sigma_{\rm X}^{\rm spont} \tag{12}$$

where the cross sections $d^2\sigma_{\rm eX}^{\rm spont}$ and $d\sigma_{\rm X}^{\rm spont}$ are those of ordinary bremsstrahlung. This is a typical result for low laser frequency regime, first described in the case of laser induced processes [15, 16]. The last equations should apply for high energy electrons, low laser frequencies and not very intense field. Nevertheless, we have to keep in mind that approaching the end of the spectrum the electrons have less and less energy, so the Born approximation will fail.

The situation investigated in more detail [4] is that of very intense laser fields for which the quiver velocity of the electron, $v_q = \omega_{\rm L}\alpha_0$, is much larger than its velocity in the absence of the laser field. This implies

$$\xi \equiv \frac{v_q}{v} = \frac{\mid e \mid \, A_0}{p} \gg 1\,. \tag{13}$$

In this regime, Karapetian and Fedorov have predicted the existence of a resonant behaviour for frequencies $\omega_{\rm X}$ in the vicinity of multiples of $\omega_{\rm L}$. In the case of the geometry $\mathbf{p}_i \perp \mathbf{s}_{\rm L}$, the resonances are single peaks located at $\omega_{\rm X} = N\,\omega_{\rm L}$ (N a pozitive integer). They have a double-peak structure in the case $\mathbf{p}_i \parallel \mathbf{s}_{\rm L}$, the two maxima being located at $\omega_{\rm X} \approx (1 \pm 1/\xi)\,N\,\omega_{\rm L}$. The predictions are based on the behaviour of the Bessel functions in Eq. (9) and are confirmed by a few numerical calculations [17]. In the conditions of Fig. 6 of [17] the oscillatory behaviour is present from low frequencies up to the tip.

We have computed the laser modified bremstrahlung spectrum, by integrating the double differential cross section (9) over the electron direction. The numerical challenge comes from the evaluation of Bessel functions of both large order and arguments. A large number of partial cross sections contribute. For instance, at the incident energy T_i=1 keV, the frequency $\omega_L = 4.68$ eV and the value $\alpha_0 = 20$ a.u., this number ranges from 700 at low frequencies up to 100 toward the end of the spectrum, for a truncation error less than 10^{-5} . In the following we have considered only the geometry $\mathbf{p}_i \parallel \mathbf{s}_{\rm L}$. In this case, the axial symmetry allows the reduction of the double integral over electron directions to a single integral. All calculations are done for atomic number $Z = 1$.

Figures 1 and 2 correspond to incident electron energy T_1= 1 keV, the frequency $\omega_L = 4.68$ eV and several values of α_0. The quantity represented is

$$\sigma = \omega_{\rm X} \left(\frac{v_i}{c}\right)^2 \frac{d\sigma_{\rm X}}{d\omega_{\rm X}}\,. \tag{14}$$

In Fig. 1 the abscissa is the ratio

$$k \equiv \frac{\hbar\omega_{\rm X}}{T_i}\,. \tag{15}$$

In the absence of the laser the spectrum ends strictly at $k = 1$, reaching the value zero. The spectra appear as smooth. The external field has two visible

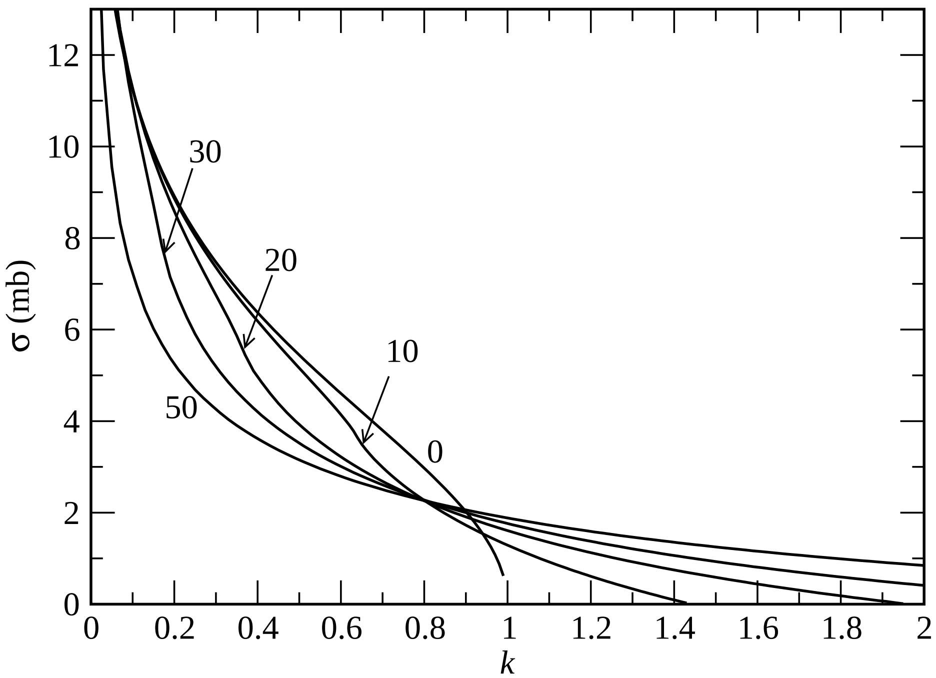

Figure 1: Coulomb bremsstrahlung spectrum [Eq. (14)] as function of the ratio $k = \hbar\omega_X/T_i$ in Born approximation, for incident electron energy $T_i = 1$ keV, laser frequency $\omega_L = 4.68$ eV. The number atached to each curve is the value of α_0 in a.u.

effects with increasing α_0: a reduction of σ at low and intermediate frequencies and an extension of the spectrum beyond the ordinary bremsstrahlung tip (3).

In Fig. 2, in another representation, taking the ratio $\omega_{\rm X}/\omega_{\rm L}$ on abscissa we illustrate the existence of the oscillations in the spectrum. In our case they are visible starting with $\alpha_0 = 50$ a.u. With increasing α_0 the distance between two succesive maxima decreases, approximately as $1 + \pi/2\xi$. This behaviour, valid with an error less than 10%, was established by noticing that the position of the n-th maximum in the spectrum is close to the position of the first maximum of the partial cross section $d\sigma_n$. At even large values ($\alpha_0 > 200$, in Fig. 2), the double peak structure predicted by Karapetyan and Fedorov comes into play. In our case the two peaks near $\omega_{\rm X} = \omega_{\rm L}$ are clearly visible for $\alpha_0 = 240$. Finally, we remark that Figure 2 covers only the small frequency region of the spectrum. A calculation at larger $\omega_{\rm X}$ shows that the oscillatory behaviour have the tendency to disappear.

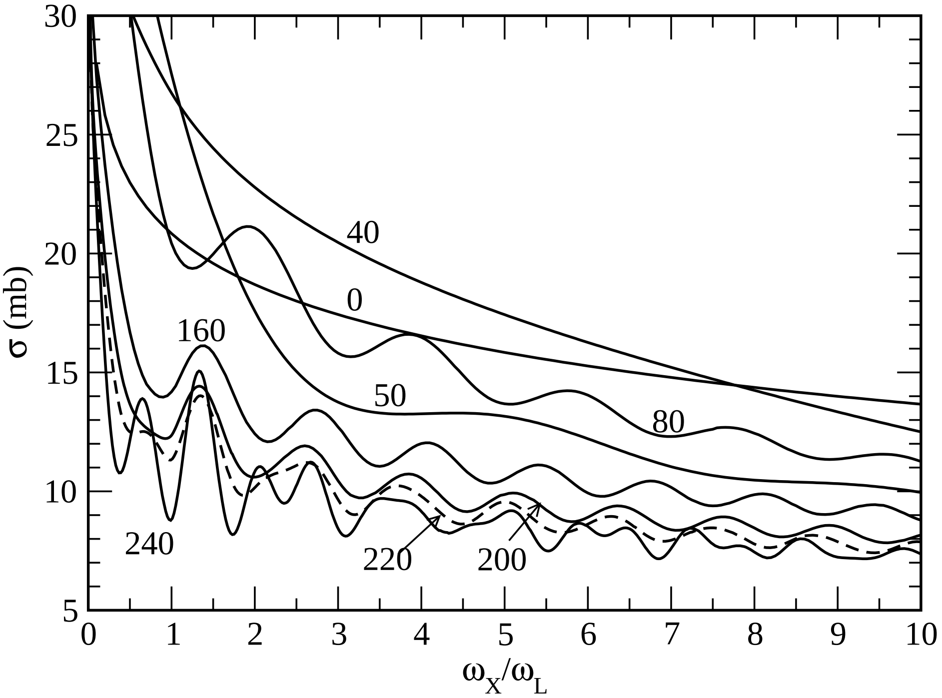

Figure 2: The same as Fig. 1, but the abscissa is now ω_X/ω_L.

6 Low laser frequency calculations

The bremsstrahlung in laser-assisted scattering was investigated by Zhou and Rosenberg [5] for the case of an *intense, low-frequency* laser field. They consider a short range potential, and a linearly polarized laser field described by the vector potential

$$\mathbf{A} = A_0\, \mathbf{s}_{\mathrm{L}} \cos \omega\, t\,. \tag{16}$$

Zhou and Rosenberg have derived two expressions for the cross section differential with respect to the photon frequency ω_X. We refer only to the expression corresponding to Eq. (3.22) of [5]. We transcribe it in a compact manner as

$$\frac{d\sigma^{\mathrm{ZR}}}{d\omega_{\mathrm{X}}} = \int_0^{\pi} \frac{d\theta}{\pi}\, \frac{p(\theta)}{p_i}\, \frac{d\sigma^{\mathrm{spont}}}{d\omega_{\mathrm{X}}}(T(\theta), \omega_X)\,, \tag{17}$$

with $d\sigma^{\mathrm{spont}}/d\omega_{\mathrm{X}}$ the corresponding differential cross section of the ordinary bremsstrahlung. The laser modified momentum $\mathbf{p}(\theta)$ has the expression

$$\mathbf{p}(\theta) = \mathbf{p}_i - e\, A_0\, \cos\theta\, \mathbf{s}_{\mathrm{L}}\,. \tag{18}$$

The final energy for the cross section under the integral sign depends on θ, according to

$$T_f = T(\theta) - \hbar\,\omega_{\rm X}\,, \qquad T(\theta) = \frac{p^2(\theta)}{2m_e}\,. \tag{19}$$

As emphasised by its authors [5], the essential feature of the result is the connection of the laser modified cross section with the cross section in the absence of the field: the former is an average of the latter, multiplied by the ratio $p(\theta)/p_i$, over the phase of the laser field.

We shall discuss now some consequences of Rosenberg and Zhou equation (17).

We analyze the condition $T_f \geq 0$, i.e., the inequality

$$\hbar\omega_X \leq \frac{(\mathbf{p}_i - e\mathbf{A}_0\,\cos\theta)^2}{2m_e}\,, \tag{20}$$

which shows that the maximum frequency allowed is different at different values of θ. This has to implications:

i) there is a maximum frequency, which depends on the geometry,

ii) the contribution to a given frequency $\omega_{\rm X}$ does not come always from all points in the interval $(0,\,\pi)$ of the integration variable θ in (17).

The maximum frequency allowed is

$$\omega_{\rm X}^{\rm max} = (1 + 2\,\lambda\,\xi + \xi^2)\,\omega_{\rm tip}^{\rm spont}, \qquad \lambda \equiv \left|\mathbf{s}_{\rm L}\cdot\frac{\mathbf{p}_i}{p_i}\right|\,, \tag{21}$$

and reduces to $\omega_{\rm tip}^{\rm spont}$ in the absence of the field.

Equation (21) is an analytical expression for the tip position in the presence of the field. The shift of the tip depends on the ratio ξ and on the direction of the incident electron momentum with respect to the laser polarization. In particular, for incident electrons parallel to the laser polarization ($\lambda = 1$), one has

$$\omega_{\rm X}^{max} - \omega_{\rm tip}^{\rm spont} = \xi(2+\xi)\,\omega_{\rm tip}^{\rm spont}\,. \tag{22}$$

We report our conclusions concernig the integration domain in θ in the case $\lambda = 1$.

1) The vicinity of the maximum value is associated to the vicinity of the point $\theta = 0$.

2) If $k \leq (1-\xi)^2$, the integration domain is $(0,\pi)$ for $\xi \leq 1$ and $(0,\theta_1) \cup (\theta_2,\pi)$ for $\xi > 1$, where

$$\cos\,\theta_1 = -\frac{1-\sqrt{k}}{\xi}\,, \qquad \cos\,\theta_2 = -\frac{1+\sqrt{k}}{\xi} \tag{23}$$

3) For $k > (1-\xi)^2$, the integration range is $(0,\theta_1)$.

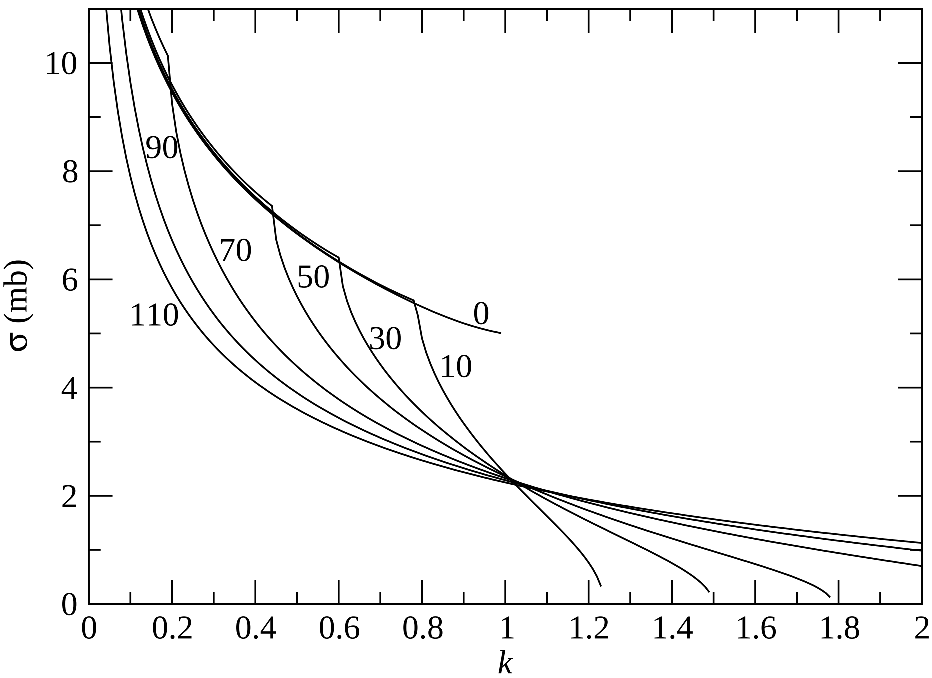

Figure 3: Coulomb bremsstrahlung spectrum [Eq. (14)] as function of k in the low laser frequency approximation, for $T_i = 0.2$ keV, and $\omega_L = 1.17$ eV.

Our numerical evaluation of the integral over θ takes care of the situation just described.

Finally, we mention that Eq. (17) does not display partial cross-sections. This expression is the equivalent of a sum rule. An estimate of the maximum number of photons absorbed is possible using the energy conservation,

$$n_{max} = \frac{\omega_{\rm X}^{\rm max} - \omega_{\rm tip}^{\rm spont}}{\omega_L} = (2\,\lambda\,\xi + \xi^2)\,\frac{T_i}{\hbar\omega_{\rm L}}\,. \tag{24}$$

This number is huge for incident electron energy in the keV range and laser photon energy in the eV range, but the corresponding contribution is completely negligible.

We made numerical evaluations of the laser modified bremsstrahlung spectrum based on Eq. (17) for the case of the Coulomb potential, using Sommerfeld's [18] formula for $d\sigma_{\rm X}^{\rm spont}$, for different incident electron energies, laser frequencies and intensities.

Figure 3 corresponds to an incident electron energy $T_i = 0.2$ keV, laser frequency $\omega_{\rm L} = 1.17$ eV and several values of α_0, chosen such that $\xi \leq 1$. The extent of the spectrum beyond the ordinary bremsstrahlung tip is visible for $\alpha_0 \neq 0$. At some value of the abscissa, a sudden modification of the tangent is

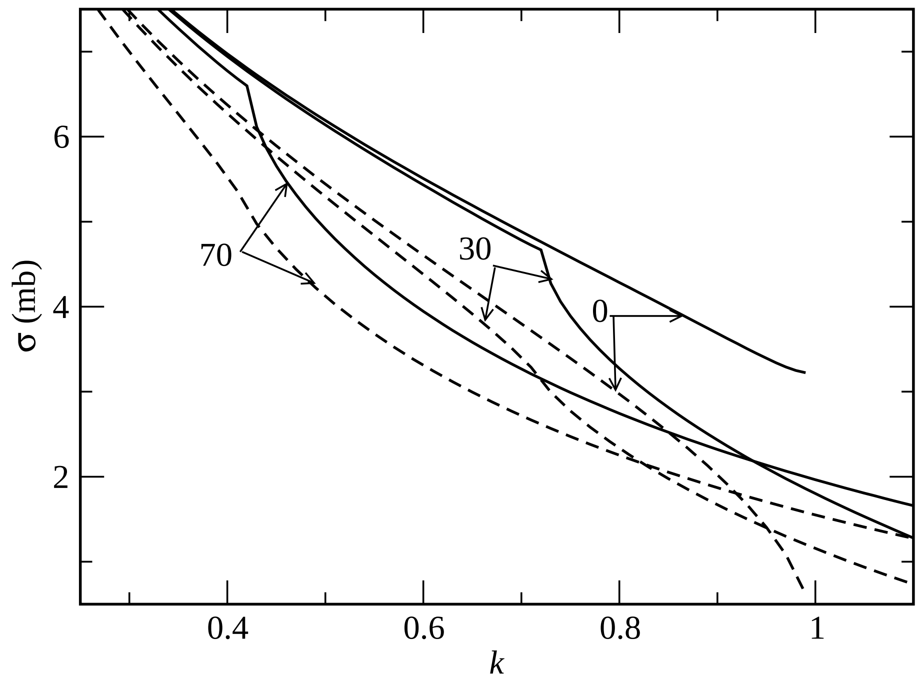

Figure 4: Comparison of the low laser frequency approximation (continuous line) with Born approximation (dashed line) for the Coulomb bremsstrahlung spectrum [Eq. (14)] as function of k, for $T_i = 1$ keV and $\omega_L = 1.17$ eV.

visible on some of the graphs. This (somehow) surprising behaviour appears exactly at the emitted frequency for which the interval in θ which contributes to the spectrum begins to decrease, according to the rules described previously. At the same time, we notice that for lower values of $\omega_{\rm X}$ the spectrum is not much modified by the increase of α_0. The situation is explored in more detail in Fig. 4, for $T_i = 1$ keV and $\omega_L = 1.17$ eV. In this figure, we compare the spectra obtained in the low frequency approximation (17), using Sommerfeld formula, with those in Born approximation (9), represented by dashed lines, for three values of α_0. We observe that to the rapid variation of the spectrum, described before, there corresponds a sinuous variation in Born approximation (visible also in Fig. 1). Also, it is to be remarked that the valability of the Born approximation is not worsened by increasing α_0. In the absence of the field the value of the spectrum at the tip is zero in Born approximation. But, as noticed previously, Born approximation is not valid in the tip region. The difference between Born and exact Coulomb in the presence of the laser field is comparable with that in the absence of the field. The extent of the spectrum beyond the ordinary bremsstrahlung tip is visible in both Figs. 3 and 4.

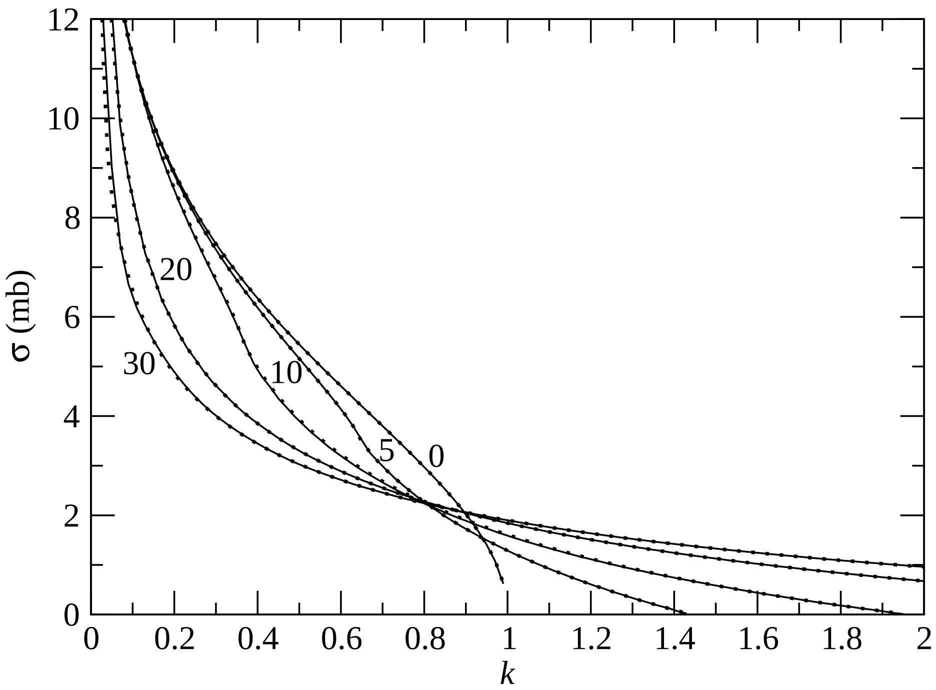

Figure 5: Born approximation results for the spectrum [Eq. (14)]: with Eq. (9) - continuous line and with Eq. (17) - dotted line, as function of k, at $T_i =$ 1 keV and $\omega_L = 9.36$ eV.

7 Other quantitative results

As a check of our calculation, but not only for that purpose, we have compared the spectra coming from Karapetian and Fedorov equation (9) with those from Zhou and Rosenberg equation (17) in which the Coulomb bremsstrahlung spectrum was taken in Born approximation. In Fig. 5, the comparison is done for $T_i = 1$ keV, $\omega_L = 9.36$ eV, and several values of α_0 : 0, 5, 10, 20 and 30 a.u. We see that the two calculations produce almost the same results. This was important in order to gain confidence in our results, as the used equations are very different, as are the methods required for their evaluation. With the increase of α_0 there is a slight tendency of amplification of the difference between the two calculations. This is not surprising, because Zhou and Rosenberg [5] have argued that the low frequency approximation is expected to fail at laser intensities for which ξ is approaching 1.

In our opinion, the study of the modifications brought by the presence of a laser field on the spontaneous emission of radiation by an electron can be continued in several directions, based on more realistic descriptions of both the atomic target and the external electromagnetic field. It remains also to be seen if the sudden change in the spectrum displayed in Figs. 3 and 4 is connected with the adopted approximation.

References

[1] C. J. Joachain, M. Dörr, and N. K. Kylstra, Adv. At. Mol. Phys. **42**, 225 (2000).

[2] R. H. Pratt and L. J. Feng, in *Atomic Inner Shell Physics*, edited by B. Crasemann (Plenum Press, New York, 1985), p. 533.

[3] F. Ehlotzky, A. Jaron, and J. Z. Kaminski, Phys. Rep. **297**, 64 (1998).

[4] P.V. Karapetyan and M. V. Fedorov, Zh. Eksp. Teor. Fiz. **75**, 816 (1978) [Sov. Phys. JETP **48**, 412 (1979).

[5] F. Zhou and L. Rosenberg, Phys. Rev. A **48**, 505-515 (1993).

[6] A. V. Borisov and V. Ch. Zhukovskii, Sov. Phys. JETP **43**, 247 (1976).

[7] V. P. Krainov, J. Phys. B **33**, 1585 (2000).

[8] V. P. Krainov and S.P. Roshchupkin, Sov. Phys. JETP **57**, 754 (1983).

[9] M. Gavrila, A. Maquet and V. Véniard, Phys. Rev. A **32**, 2537 (1985).

[10] V. Florescu and V. Djamo, Phys. Lett. A **119**, 73 (1986).

[11] M. Gavrila, A. Maquet and V. Véniard, Phys. Rev. A **42**, 236 (1990).

[12] A. A. Krylovetskii, N. L. Manakov, S. I. Marmo, and A. F. Starace,JETP **95**, 1006 (2002).

[13] A. Florescu and V. Florescu, Phys. Lett. A **226**, 280-288 (1997).

[14] A. Florescu and V. Florescu, Phys. Rev. A **61**, 033406-1 (2000).

[15] N. M. Kroll and K. M. Watson, Phys. Rev. A **8**, 804 (1973).

[16] F. V. Bunkin and M. V. Fedorov, Sov. Phys. JETP **22**, 844 (1966).

[17] M. V. Fedorov and M. Yu. Ivanov, Laser Physics, **3**, 365 (1993).

[18] A. Sommerfeld, *Atombau und Spektrallinien* (Viewig, Braunschweig, 1939), Vol. 2.