

*Dedicated to Prof. Dorin N. Poenaru's  
70th Anniversary*

## TWO CENTER SHELL MODEL WITH WOODS-SAXON POTENTIALS

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*Abstract.* Single-particle energies and wave functions of an axially two-center Woods-Saxon potential are computed. The Hamiltonian includes the spin-orbit interaction. The nuclear shape is determined by two ellipsoids smoothly joined with a neck region.

*Key words:* nuclear models, two-center shell models, woods-Saxon potentials.

### 1. INTRODUCTION

The theoretical study of binary disintegration processes is limited by the difficulties encountered in the calculations of single-particle levels for very deformed one-center potentials. Indeed, on one hand, central potentials are not able to describe in a correct manner the shapes for the passage of one nucleus to two separated nuclei and, on other hand, for very large prolate deformations the sum of single-particle energies obtained from the level scheme reaches an infinite value. These difficulties are surpassed by considering that the mean field is generated by nucleons moving in a double center potential. A two-center model allows the description of single-particle energy evolutions from the ground-state up to the formation of two separated fragments [1, 2] of a dinuclear system. Most of the applications of this method are based on a double oscillator potential [3] and more recently, the single-particle motion in fusing systems and the peripheral collisions were treated employing Wood-Saxon wells [4] with the potential expansion method.

A new version of the two-center model, namely the superasymmetric two center shell model (STCSM) was developed [5–7] by the author and was used to explain the fine structure in alpha- and cluster decays [6, 8].

The renewal interest for the fission physics determined the use of the STCSM in the analysis of the role played by the single-particle states during the tunneling of the double barrier. Information about the role of the dissipation and effects due to the dynamics were revealed [9–11]. Investigating fission processes, unfortunately, very large values of the heights of the double barrier were obtained. A cause for this behavior was presumed to be due to the intrinsic limitations of the modified oscillator model [7]. The angular momentum interaction constants are determined only to fit several single particle levels in the vicinity of the Fermi energy and are not appropriate for all the levels taken into account when the shell effects are computed. On an other hand, the spin-orbit operator cannot take into account a realistic deformed mean field potential. In the following, a improved version of a two center shell model is presented that takes into account Woods-Saxon potentials.

## 2. THE NUCLEAR SHAPE PARAMETRIZATION

Two ellipsoids of semiaxis  $a_i, b_i$  ( $i = 1, 2$ ) are smoothly joined by an intermediate surface that defines the neck region. The intermediate surface is obtained by the rotation of a circle of curvature  $C = S/R_3$  around the axis of symmetry. In Fig. 1, the shape is displayed and its parameters can be identified. The parametrization of the nuclear surface in cylindrical coordinates has the next form:

$$\rho = \begin{cases} b_1 \sqrt{1 - \left(\frac{z-c_1}{a_1}\right)^2}, & z \leq z_1, \\ \rho_3 - S \sqrt{R_3^2 - (z - c_3)^2}, & z_1 < z < z_2, \\ b_2 \sqrt{1 - \left(\frac{z-c_2}{a_2}\right)^2}, & z_2 \leq z, \end{cases} \quad (1)$$

where  $c_i$  ( $i = 1, 2$ ) are the positions of the two ellipsoid centers and  $c_3$  characterizes the center of the circle that defines the neck. This nuclear shape parametrization depends on five degrees of freedom, namely, the two deformations of the fragments, determined by the eccentricities  $e_i = \sqrt{1 - \left(\frac{b_i}{a_i}\right)^2}$ , the necking characterized by the curvature  $C$ , the elongation measured by  $R$ , the internuclear distance between  $c_1$  and  $c_2$ , and the mass asymmetry that can be determined, for instance, as the ratio of the volumes of the two fragments. Within this parametrization, it is possible to construct swollen shapes in the median surface when the parameter  $S = -1$  and necked ones when  $S = 1$ . If the eccentricities  $e_i$ , the elongation  $R$ , the curvature  $C$  and the mass numbers of the nascent fragments  $A_1$  and  $A_2$  are known, it is straightforward to obtain all the geometrical parameters that define the shape by solving the equation

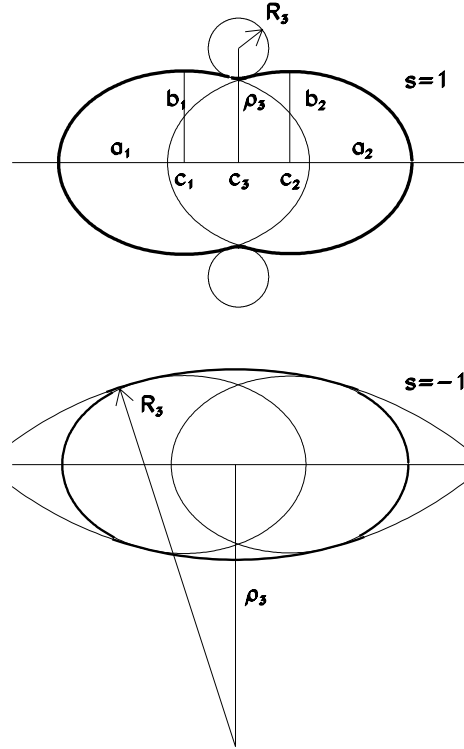


Fig. 1 – Nuclear shape parametrization: two ellipsoids smoothly joined with a neck region. When  $S=1$ , the median surface is necked while when  $S = -1$ , the median surface is swollen.

for the volume conservation. From now on, the index 1 will assign the heavy fragment while the index 2 is reserved to the the light one. The parent nucleus has the index 0.

This nuclear shape parametrization can describe in a good approximation all the nuclear shapes encountered in fission processes.

### 3. THE WOODS-SAXON POTENTIAL

The mean field potential is defined in the following way [12]:

$$V(\rho, z) = -\frac{V_0}{1 + \exp\left[\frac{\Delta(\rho, z)}{a}\right]}, \quad (2)$$

where  $\Delta(\rho, z)$  represents the distance (taken with minus sign inside the nucleus) between a point  $(\rho, z)$  and the nuclear surface. This distance is measured only along the normal direction on the surface.  $V_0$  is the depth of

the potential while  $a$  is the diffuseness parameter. In our work, the depth is  $V_0 = V_c[1 \pm \kappa(N - Z)/(N + Z)]$  with plus sign for protons and minus sign for neutrons,  $V_c = 51$  MeV,  $a = 0.67$  fm,  $\kappa = 0.67$ .

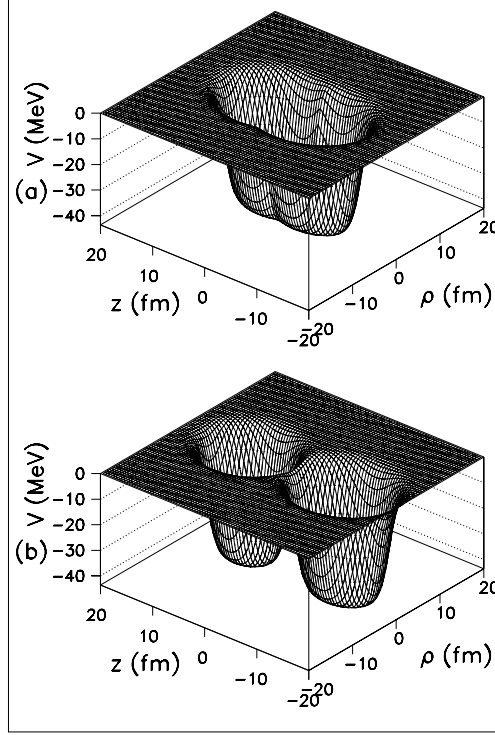


Fig. 2 – Mean field Woods-Saxon potential as function of the cylindrical coordinates  $\rho$  and  $z$  for different values of the elongation. One elongation  $R = 7$  fm (a) characterizes overlapped nascent fragments while the second one  $R = 18$  fm (b) gives rise to two separated fragments.

In Fig. 2, the mean field potential is plotted as function of  $\rho$  and  $z$  for different distances between the centers and a curvature  $C = 0.5$  fm $^{-1}$ . The parameters are selected so that the fission reaction of the parent  $^{235}\text{U}$  is simulated.

The spin-orbit coupling is assumed in the form

$$V_{ls} = -\lambda \left( \frac{1}{2mc} \right)^2 (\nabla V \times \vec{p}) \vec{s}, \quad (3)$$

where  $\lambda = 35$  is a dimensionless coupling constant,  $m$  is the nucleon mass while  $c$  denotes the speed of the light.

The spherical components of the operator

$$L = \nabla V \times p \quad (4)$$

in cylindrical coordinates are

$$L^\pm = \mp \hbar e^{\pm i\varphi} \left( \frac{\partial V}{\partial \rho} \frac{\partial}{\partial z} - \frac{\partial V}{\partial z} \frac{\partial}{\partial \rho} \pm i \frac{\partial V}{\partial z} \frac{1}{\rho} \frac{\partial}{\partial \varphi} \right) \quad (5)$$

$$L_z = i\hbar \frac{\partial V}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \varphi} \quad (6)$$

so that

$$Ls = \frac{1}{2}(L^+s^- + L^-s^+) + L_zs_z. \quad (7)$$

The last formula is suitable for calculations involving cylindrical coordinates.

The next step is to obtain the solutions of the Schrodinger equation

$$\left[ -\frac{\hbar^2}{2m}\Delta + V_0(\rho, z) + V_{ls}(\rho, z) \right] \Psi(\rho, z, \varphi) = E\Psi(\rho, z, \varphi). \quad (8)$$

No analytical solutions can be found for such potentials. A suitable eigenvector basis able to diagonalize the Woods-Saxon potential can be obtained with the double center harmonic oscillator model.

#### 4. THE SEMI-SYMMETRIC TWO-CENTER OSCILLATOR

A complete analytical eigenvector basis can be only obtained for the semi-symmetric two-center oscillator. This potential corresponds to a shape parametrization given by two ellipsoids that possess the same semi-axis perpendicular on the axis of symmetry as plotted in Fig. 3.

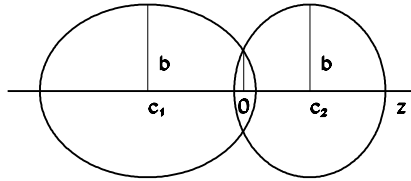


Fig. 3 – Nuclear shape parametrization for the semi-symmetric two-center shell model based on oscillators. Two intersected ellipsoids with the same semiaxis  $b$  perpendicular on the axis of symmetry are taken into account to determine the analytic eigenvector basis. The origin on the  $z$ -axis corresponds to the plane of intersection of the ellipsoids.

The potential is

$$V_s(\rho, z) = \begin{cases} \frac{1}{2}m\omega_{z1}^2(z - c_1)^2 + \frac{1}{2}m\omega_\rho^2, & z < 0, \\ \frac{1}{2}m\omega_{z2}^2(z - c_2)^2 + \frac{1}{2}m\omega_\rho^2, & z \geq 0, \end{cases} \quad (9)$$

where  $\omega$  denotes the stiffness of the potential along different directions as follows,  $\omega_{z1} = \omega_0 \frac{R_0}{a_1}$ ,  $\omega_{z2} = \omega_0 \frac{R_0}{a_2}$ ,  $\omega_\rho = \omega_0 \frac{R_0}{b_1}$ ,  $\omega_0 = 41A_0^{-1/3}$ ,  $R_0 = 1.16A_0^{1/3}$ , in order to ensure a constant value of the potential on the surface. The origin on the  $z$ -axis is considered the location of the plane of intersection between the two ellipsoids.

An analytic system of eigenvectors can be obtained for  $V_0$  by solving the Schrödinger equation:

$$\left[ -\frac{\hbar^2}{2m_0} + V_0(\rho, z) \right] \Psi(\rho, z, \varphi) = E\Psi(\rho, z, \varphi). \quad (10)$$

The analytic solution of Eq. (10) is obtained using the ansatz

$$\Psi(\rho, z, \varphi) = Z(z)R(\rho)\Phi(\varphi), \quad (11)$$

with

$$\Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} \exp(im\varphi) \quad (12)$$

$$R_{nm}(\rho) = \sqrt{\frac{2n!}{(n+m)!}} \alpha_{\rho 1} \exp\left(-\frac{\alpha_{\rho 1}^2 \rho^2}{2}\right) (\alpha_{\rho 1} \rho)^m L_n^m(\alpha_{\rho 1}^2 \rho^2) \quad (13)$$

$$Z_\nu(z) = \begin{cases} C_{\nu_1} \exp\left(-\frac{\alpha_{z1}^2(z-c_1)^2}{2}\right) \mathbf{H}_{\nu_1}[-\alpha_{z1}(z+c_1)], & z < 0, \\ C_{\nu_2} \exp\left(-\frac{\alpha_{z2}^2(z-c_2)^2}{2}\right) \mathbf{H}_{\nu_2}[\alpha_{z2}(z-c_2)], & z \geq 0, \end{cases} \quad (14)$$

where  $L_n^m(x)$  is the Laguerre polynomial,  $\mathbf{H}_\nu(\zeta)$  is the Hermite function,  $\alpha_i = (m_0\omega_i/\hbar)^{1/2}$  ( $i = z1, z2, \rho$ ) are length parameters, and  $C_{\nu_i}$  denote the normalization constants. The quantum numbers  $n$  and  $m$  are integers while the quantum number  $\nu$  along the  $z$ -axis is real and has different values for the intervals  $(-\infty, 0]$  and  $[0, \infty)$ . Imposing conditions for the continuity of the wave function and its derivative, together with those for the stationary energy and orthonormality, the values of  $\nu_1$ ,  $\nu_2$ ,  $C_{\nu_1}$  and  $C_{\nu_2}$  are obtained. Details concerning these solutions and expressions for the normalization constants are found in Ref. [5]. For reflection-symmetric shapes, the solutions along the  $z$ -axis are also characterized by the parity as a good quantum number [13].

## 5. RESULTS AND DISCUSSION

An example of a two-center diagram with Woods-Saxon potentials is given. The Woods-Saxon two-center shell model described in this work was used to compute single-particle energies as function of the internuclear distance  $R$  for the reaction  $^{26}\text{Al} \rightarrow ^{12}\text{C} + ^{14}\text{N}$ . The neutron single-particle level scheme is plotted in Fig. 4.

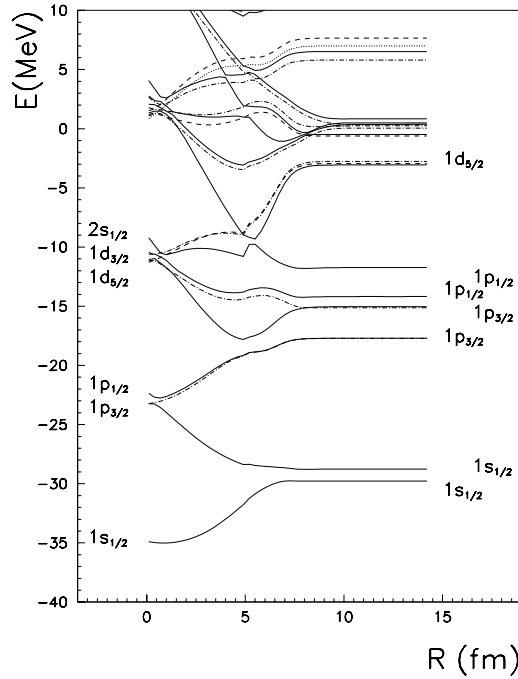


Fig. 4 – Neutron single-particle energy diagram as function of the elongation  $R$  obtained with the double-center Woods-Saxon model. The major quantum number is considered as  $N_{max} = 5$ . The reaction  $^{26}\text{Al} \rightarrow ^{12}\text{C} + ^{14}\text{N}$  is treated.

Due to the axial symmetry of the system, the good quantum numbers are the projection on the  $z$ -axis of the nucleon spin  $\Omega$ . The solid lines are for  $\Omega = 1/2$ , the dash-dotted lines are for  $\Omega = 3/2$ , the dashed line for  $\Omega = 5/2$  and the dotted line for  $\Omega = 7/2$ . For  $R = 0$ , the level scheme of the spherical  $^{26}\text{Al}$  parent is obtained. The curvature is  $C = 0.5 \text{ fm}^{-1}$  and the eccentricities are  $e_i = 0$  ( $i = 0, 1, 2$ ). When the elongation  $R$  increases, two separated fragments are formed, afterwards the parent is splitted in two nuclei, and for very large values of  $R$ , the two levels schemes of the individual nuclei that are formed during the disintegration become superimposed. The spectroscopic factors are

marked on the figure for the initial parent considered spherical and for each of the two fragments in order to identify the levels.

In principle, a two-center diagram can be used to investigate all reactions that involve two separated nuclei. On one hand, it is possible to predict which single-particle inelastic excitations and neutron transfer reaction will be enhanced in heavy ion collisions. On another hand, it is also possible to determine the transition levels for the fission reactions, leading to a improved calculation of the fission cross-sections. It is customary for evaluations purposes to fit the transition levels from experimental values of the cross-sections. A realistic model for the single-particle energies able to describe the passage from one nucleus in two separated fragments will provide a powerfull tool to validate the evaluations. A realistic model for the level scheme can also provide information about important quantities as the dissipation by solving the microscopic equations of motion.

The previous versions of the two-center shell model were based on a harmonic oscillator corrected with spin-orbit and  $L^2$  terms. An unrealistic increase of the potential outside the nucleus introduces a peculiar behavior in the model that cannot be avoided, so that the variations of single-particle levels that lies above the Fermi energy cannot be correctly described. This undesired behavior is eliminated in the actual version that uses a Woods-Saxon form for the potential, so that more realistic levels schemes are expected.

In one-center Woods-Saxon models [12] it is possible to speak only about two-center applications, when the shapes are determined to be very necked so that it is possible to identify the apparition of two fragments. A better simulation of two separated fragments can be obtained by taking into account a very large number multipole deformations. In our treatment, only five degrees of freedom are sufficient to treat the fission process and the neck parameter is an independent parameter.

## REFERENCES

1. P. Holzer, U. Mosel and W. Greiner, Nucl. Phys., A **138**, 241 (1969).
2. J. Maruhn and W. Greiner, Z. Phys., **251**, 431 (1972).
3. W. Greiner, J.Y. Park and W. Scheid, in *Nuclear Molecules*, World Scientific, Singapore, 1994.
4. A. Diaz-Torres and W. Scheid, Nucl. Phys., A **757**, 373 (2005).
5. M. Mirea, Phys. Rev., C **54**, 302 (1996).
6. M. Mirea, Phys. Rev., C **57**, 2484 (1998).
7. M. Mirea, Nucl. Phys., A **780**, 13 (2006)
8. M. Mirea, Phys. Rev., C **63**, 034603 (2001).
9. M. Mirea, L. Tassan-Got, C. Stephan, C.O. Bacri and R.C. Bobulescu, Europhys. Lett., **73**, 705 (2006).

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10. M. Mirea, L. Tassan-Got, C. Stephan, C.O. Bacri, P. Stoica and R.C. Bobulescu, *J. Phys., G* **31**, 1165 (2005).
  11. M. Mirea, L. Tassan-Got, C. Stephan and C.O. Bacri, *Nucl. Phys., A* **735**, 21 (2004).
  12. S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski and T. Werner, *Comput. Phys. Comm.*, **46**, 37 (1987).
  13. E. Badraxe, M. Rizea and A. Sandulescu, *Rev. Roum. Phys.*, **19**, 63 (1974).