

A MATHEMATICAL MODEL FOR KINEMATIC LINKAGES MADE OF DEFORMABLE COMPOSITE BARS BUILT BY USING THE HAMILTON'S VARIATION PRINCIPLE

SABIN RIZESCU, MARCELA URSACHE, ELENA TAINA AVRAMESCU, DUMITRU BOLCU

University of Craiova, 13 A.I. Cuza Street, Craiova, Romania

(Received April 26, 2007)

Abstract. This paper consists in building up a new perspective concerning some analytical approach of modeling the behavior of kinematic linkages made of linear-elastic deformable bars. We have been focused on case of composite orthotropic bars having elastic symmetry.

Key words: spatial linkages, deformable elements, composite bars, elastic symmetry, new HSDT deformation hypothesis.

1. INTRODUCTION

Describing the elasto-dynamics of composite bars by using the Classic Theory of Elasticity is practically impossible due to the fact that the Bernoulli hypothesis simply does not work in case of composite bars. Composite materials are generally non-homogenous and non-isotropic [4, 5]. For such kind of materials new methods of study have been developed [11]. This paper proposes a new HSDT deformation theory [11, 12] which is to be used to kinematic linkages made of deformable composite bars.

2. THE MECHANICAL WORK OF THE EXTERNAL CHARGES

We shall consider that on the j bar axis are continuously distributed the next linear specific force:

$$\vec{p}_j = \{p^j\}^t \left\{ \vec{i}^j \right\}, \quad (1)$$

the linear specific couple:

$$\vec{m}_j = \{m^j\}^t \left\{ \vec{i}^j \right\}. \quad (2)$$

Due to the fact that the Gay conditions are fully satisfied, we can write the expression of the mechanical work of the external charges:

$$L_e = \sum_{j=1}^n \int_{0_j}^{L_j} \left(\{p^j\}^t \{u^j\} + \{m^j\}^t \{\theta^j\} \right) dx_1^j. \quad (3)$$

We shall note “0_j” the boundary point in which is to be placed the origin O_j of the own reference system of j bar, the second boundary point will be obviously L_j , where L_j is the length of the j bar. In our holonomic case we have:

$$\delta \int_{t_0}^{t_1} L_e dt = \int_{t_0}^{t_1} \delta L_e dt = \int_{t_0}^{t_1} \left(\sum_{j=1}^n \int_{0_j}^{L_j} \left(\{p^j\}^t \{\delta u^j\} + \{m^j\}^t \{\delta \theta^j\} \right) dx_1^j \right) dt. \quad (4)$$

Concentrated forces and concentrated couples are not to be considered.

3. THE MATHEMATICAL MODEL FOR THE VIBRATIONS OF EACH BAR OF A LINKAGE MADE OF DEFORMABLE COMPOSITE BARS

For the kinetic energy which has been calculated in [13] we will further apply the δ operator in order to obtain an even more simple form for the so-called “differentiated” kinetic energy. Considering all kinds of symmetries described in [13] and [14] we finally obtain:

$$\begin{aligned} \delta \int_{t_0}^{t_1} T dt &= \sum_{j=1}^n \int_{t_0}^{t_1} \int_{0_j}^{L_j} \iint_{S_j} \left\{ \left[-\dot{A}^j \right]^t \left[{}^0S^j \right]^t - 2 \left[\dot{u}^j \right]^t \left[{}^0\Omega^j \right] - \left\{ \{x^j\}^t + \{u^j\}^t \right\} \cdot \right. \\ &\cdot \left[\left[{}^0\dot{\Omega}^j \right] + \left[{}^0\Omega^j \right]^2 \right] - \left\{ \ddot{u}^j \right\}^t \left\{ \delta u^j \right\} \cdot \rho_j dS_j dx_1^j dt + \sum_{j=1}^n \int_{t_0}^{t_1} \int_{0_j}^{L_j} \iint_{S_j} \left\{ 2 \left\{ \dot{\theta}^j \right\}^t \left[r_j \right] \cdot \right. \\ &\cdot \left[{}^0\Omega^j \right] + \left\{ \left\{ r_j \right\}^t + \left\{ \dot{\theta}^j \right\}^t \left[r_j \right] \right\} \cdot \left[\left[{}^0\dot{\Omega}^j \right] + \left[{}^0\Omega^j \right]^2 \right] + \left\{ \ddot{\theta}^j \right\}^t \left[r_j \right] \left\{ \left[r_j \right] + \right. \\ &+ \left. \left\{ -2 \left\{ \dot{\theta}^j \right\}^t \left[Q_j \right]^t \left[{}^0\Omega^j \right] - \left\{ \theta^j \right\}^t \left[Q_j \right]^t \left[\left[{}^0\dot{\Omega}^j \right] + \left[{}^0\Omega^j \right]^2 \right] - \left\{ \ddot{\theta}^j \right\}^t \left[Q_j \right]^t \right\} \cdot \right. \\ &\left. \left[Q_j \right] \left\{ \delta \theta^j \right\} \cdot \rho_j dS_j dx_1^j dt. \right. \end{aligned} \quad (5)$$

The Hamilton's variation principle is about the extreme of the next functional:

$$S = \int_{t_0}^{t_1} (T - L_d + L_e) dt, \quad (6)$$

considered with its own synchronic conditions: $\delta \overline{w^j}(t_0) = \delta \overline{w^j}(t_1) = \vec{0}$.

Let's put (36) of [13], (44) of [14] and (4) in (6) and taking into account that the synchronic conditions are to be satisfied which means that $\{\delta u^j\}$ and $\{\delta \theta^j\}$ are to have arbitrary values. In such a case the expression being in a position to multiply them are to have zero value. So it means that, for the expression related to $\{\delta u^j\}$:

$$\begin{aligned} & \iint_{S_j} \left\{ \left\{ -\dot{A}^j \right\}^t \left[{}^0S^j \right]^t - 2 \left\{ \dot{u}^j \right\}^t \left[{}^0\Omega^j \right] - \left\{ x^j \right\}^t + \left\{ u^j \right\}^t \right\} \left[\left[{}^0\dot{\Omega}^j \right] + \left[{}^0\Omega^j \right]^2 \right] - \\ & - \left\{ \ddot{u}^j \right\}^t \left\{ \rho_j dS_j + \iint_{S_j} \left\{ \left\{ u^j \right\}_{,11}^t \left[E_j^1 \right]^t + \left\{ \theta^j \right\}_{,1}^t \left[\left[I_2^j \right] \left[E_j^2 \right]^t - \left[Q_j \right]_{,2}^t \left[E_j^2 \right]^t + \right. \right. \right. \\ & \left. \left. \left. + \left[I_3^j \right] \left[E_j^3 \right]^t - \left[Q_j \right]_{,3}^t \left[E_j^3 \right]^t \right\} \right\} dS_j + \left\{ p^j \right\}^t = \{0\}^t. \end{aligned} \quad (7)$$

And, for the expression related to $\{\delta \theta^j\}$:

$$\begin{aligned} & \iint_{S_j} \left\{ \left\{ 2 \left\{ \dot{\theta}^j \right\}^t \left[r_j \right] \left[{}^0\Omega^j \right] + \left\{ \left\{ r_j \right\}^t + \left\{ \theta^j \right\}^t \left[r_j \right] \right\} \cdot \left[\left[{}^0\dot{\Omega}^j \right] + \left[{}^0\Omega^j \right]^2 \right] \right\} + \\ & + \left\{ \ddot{\theta}^j \right\}^t \left\{ \left[r_j \right] \right\} \left\{ \left[r_j \right] \right\} + \left\{ -2 \left\{ \dot{\theta}^j \right\}^t \left[Q_j \right]_{,3}^t \left[{}^0\Omega^j \right] - \left\{ \theta^j \right\}^t \left[Q_j \right]_{,2}^t \left[\left[{}^0\dot{\Omega}^j \right] + \left[{}^0\Omega^j \right]^2 \right] \right\} - \\ & - \left\{ \ddot{\theta}^j \right\}^t \left\{ \left[Q_j \right]_{,3}^t \right\} \left\{ \left[Q_j \right]_{,3}^t \right\} \rho_j dS_j - \iint_{S_j} \left\{ \left\{ \theta^j \right\}_{,11}^t \left[r_j \right] \left[E_j^1 \right]^t + \left\{ \theta^j \right\}_{,1}^t \left[-\left[Q_j \right]_{,2}^t \left[E_j^2 \right]^t - \right. \right. \\ & \left. \left. - \left[Q_j \right]_{,3}^t \left[E_j^3 \right]^t \right\} \left[r_j \right] dS_j + \iint_{S_j} \left\{ \theta^j \right\}_{,11}^t \left[Q_j \right]_{,1}^t \left[E_j^1 \right]^t \left[Q_j \right]_{,1}^t dS_j - \iint_{S_j} \left\{ \left\{ u^j \right\}_{,1}^t \left[E_j^1 \right]^t + \right. \right. \end{aligned}$$

$$\begin{aligned}
& + \{\theta^j\}^t \left[[I_2^j] [E_j^2]^t - [Q_j]_{,2}^t [E_j^2]^t + [I_3^j] [E_j^3]^t - [Q_j]_{,3}^t [E_j^3]^t \right] dS_j - \\
& - \iint_{S_j} \left\{ -\{u^j\}_{,1}^t [E_j^1]^t + \{\theta^j\}_{,1} \left[-[r_j] [E_j^1]^t + [Q_j] [E_j^1]^t \right] \right\} [Y_3^j] [Q_j]_{,2} dS_j - \\
& - \iint_{S_j} \left\{ \theta^j \right\}^t \left[-[I_2^j] [E_j^2]^t + [Q_j]_{,2}^t [E_j^2]^t - [I_3^j] [E_j^3]^t + [Q_j]_{,3}^t [E_j^3]^t \right] \cdot \\
& \cdot [Y_3^j] [Q_j]_{,2} dS_j - \iint_{S_j} \left\{ \{u^j\}_{,1}^t [E_j^1]^t + \{\theta^j\}_{,1} \left[[r_j] [E_j^1]^t - [Q_j] [E_j^1]^t \right] \right\} \cdot \\
& \cdot [Y_2^j] [Q_j]_{,3} dS_j - \iint_{S_j} \left\{ \theta^j \right\}^t \left[[I_2^j] [E_j^2]^t - [Q_j]_{,2}^t [E_j^2]^t + [I_3^j] [E_j^3]^t - \right. \\
& \left. - [Q_j]_{,3}^t [E_j^3]^t \right] [Y_2^j] [Q_j]_{,3} dS_j + \{m^j\}^t = \{0\}^t. \tag{8}
\end{aligned}$$

Considering (19) of [13] and considering that:

$$\left[{}^0\Omega^j \right]^t = -\left[{}^0\Omega^j \right]; \left[{}^0\dot{\Omega}^j \right]^t = -\left[{}^0\dot{\Omega}^j \right] \text{ and } \left[\left[{}^0\Omega^j \right]^2 \right]^t = \left[{}^0\Omega^j \right]^2, \tag{9}$$

$$\text{and for } \rho = \text{ct.}, \text{ we have: } \iint_{S_j} [Q_j]^t [{}^0\Omega^j] [Q_j] \rho dS_j = [0]_{3 \times 3}; \tag{10}$$

The relations (7) and (8) are actually partial derivate equations having as unknowns $\{u^j\}$ and $\{\theta^j\}$. These unknowns depend only on the x_1^j coordinate and on the time t . In many cases the kinematic linkages are made of composite orthotropic bars being mass homogenous (MEGS). Let's note B_j and H_j the dimensions of the transversal section of the j bar. So, (7) and (8) are to become:

$$[P_j^1] \left\{ \ddot{u}^j \right\} + [P_j^2] \left\{ \dot{u}^j \right\} + [P_j^3] \{u^j\} + [P_j^4] \{\theta^j\}_{,1} + [P_j^5] \{u^j\}_{,11} = \{f_j\}; \tag{11}$$

$$[V_j^1] \left\{ \ddot{\theta}^j \right\} + [V_j^2] \left\{ \dot{\theta}^j \right\} + [V_j^3] \{\theta^j\} + [V_j^4] \{u^j\}_{,1} + [V_j^5] \{\theta^j\}_{,11} = \{g_j\}; \tag{12}$$

where:

$$[P_j^1] = \begin{bmatrix} \rho_j B_j H_j & 0 & 0 \\ 0 & \rho_j B_j H_j & 0 \\ 0 & 0 & \rho_j B_j H_j \end{bmatrix}; \quad (13)$$

$$[P_j^2] = \begin{bmatrix} 0 & -2\rho_j B_j H_j {}^0\omega_3^j & 2\rho_j B_j H_j {}^0\omega_2^j \\ 2\rho_j B_j H_j {}^0\omega_3^j & 0 & -2\rho_j B_j H_j {}^0\omega_1^j \\ -2\rho_j B_j H_j {}^0\omega_2^j & 2\rho_j B_j H_j {}^0\omega_1^j & 0 \end{bmatrix}; \quad (14)$$

$$[{}^0\Omega^j] = \begin{bmatrix} 0 & {}^0\omega_3^j & -{}^0\omega_2^j \\ -{}^0\omega_3^j & 0 & {}^0\omega_1^j \\ {}^0\omega_2^j & -{}^0\omega_1^j & 0 \end{bmatrix}; \quad (15)$$

is the anti-symmetrical matrix attached to the angular speed of T_j with respect to T_0 :

$${}^0\omega_{(j)}^{\rightarrow j} = {}^0\omega_1^j \vec{i}_1^j + {}^0\omega_2^j \vec{i}_2^j + {}^0\omega_3^j \vec{i}_3^j = \{ {}^0\omega_1^j; {}^0\omega_2^j; {}^0\omega_3^j \} \left\{ \vec{i}^j \right\}; \quad (16)$$

We have also the angular acceleration of T_j with respect to T_0 :

$${}^0\varepsilon_{(j)}^{\rightarrow j} = {}^0\dot{\varepsilon}_{(j)}^{\rightarrow j} = {}^0\omega_1^{\bullet j} \vec{i}_1^j + {}^0\omega_2^{\bullet j} \vec{i}_2^j + {}^0\omega_3^{\bullet j} \vec{i}_3^j = \left\{ {}^0\omega_1^{\bullet j}; {}^0\omega_2^{\bullet j}; {}^0\omega_3^{\bullet j} \right\} \left\{ \vec{i}^j \right\}; \quad (17)$$

where:

$$[{}^0\dot{\Omega}^j] = \begin{bmatrix} 0 & {}^0\omega_3^{\bullet j} & -{}^0\omega_2^{\bullet j} \\ -{}^0\omega_3^{\bullet j} & 0 & {}^0\omega_1^{\bullet j} \\ {}^0\omega_2^{\bullet j} & -{}^0\omega_1^{\bullet j} & 0 \end{bmatrix}; \quad (18)$$

$$[P_j^3] = \begin{bmatrix} -\rho_j B_j H_j \left[({}^0\omega_2^j)^2 + ({}^0\omega_3^j)^2 \right] & \rho_j B_j H_j \left[{}^0\omega_1^j \cdot {}^0\omega_2^j - {}^0\dot{\omega}_3^j \right] & \rho_j B_j H_j \left[{}^0\omega_1^j \cdot {}^0\omega_3^j + {}^0\dot{\omega}_2^j \right] \\ \rho_j B_j H_j \left[{}^0\omega_1^j \cdot {}^0\omega_2^j + {}^0\dot{\omega}_3^j \right] & -\rho_j B_j H_j \left[({}^0\omega_1^j)^2 + ({}^0\omega_3^j)^2 \right] & \rho_j B_j H_j \left[{}^0\omega_2^j \cdot {}^0\omega_3^j - {}^0\dot{\omega}_1^j \right] \\ \rho_j B_j H_j \left[{}^0\omega_1^j \cdot {}^0\omega_3^j - {}^0\dot{\omega}_2^j \right] & \rho_j B_j H_j \left[{}^0\omega_2^j \cdot {}^0\omega_3^j + {}^0\dot{\omega}_1^j \right] & -\rho_j B_j H_j \left[({}^0\omega_1^j)^2 + ({}^0\omega_2^j)^2 \right] \end{bmatrix} \quad (19)$$

where:

$$[{}^0\Omega^j] = \begin{bmatrix} -({}^0\omega_2^j)^2 - ({}^0\omega_3^j)^2 & {}^0\omega_1^j {}^0\omega_2^j & {}^0\omega_1^j {}^0\omega_3^j \\ {}^0\omega_1^j {}^0\omega_2^j & -({}^0\omega_1^j)^2 - ({}^0\omega_3^j)^2 & {}^0\omega_2^j {}^0\omega_3^j \\ {}^0\omega_1^j {}^0\omega_3^j & {}^0\omega_2^j {}^0\omega_3^j & -({}^0\omega_1^j)^2 - ({}^0\omega_2^j)^2 \end{bmatrix}; \quad (20)$$

$$[P_j^4] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{11}{9} E_{1212}^j B_j H_j \\ 0 & -\frac{11}{9} E_{1313}^j B_j H_j & 0 \end{bmatrix}; \quad (21)$$

$$[P_j^5] = \begin{bmatrix} -E_{1111}^j B_j H_j & 0 & 0 \\ 0 & -E_{1221}^j B_j H_j & 0 \\ 0 & 0 & -E_{1331}^j B_j H_j \end{bmatrix}; \quad (22)$$

$$\{f_j\} = \left\{ \begin{array}{l} p_1^j - \rho_j B_j H_j \left[a_{o_j(j)1}^0 - \left(({}^0\omega_2^j)^2 + ({}^0\omega_3^j)^2 \right) x_1^j \right] \\ p_2^j - \rho_j B_j H_j \left[a_{o_j(j)2}^0 + \left({}^0\omega_1^j {}^0\omega_2^j + {}^0\dot{\omega}_3^j \right) x_3^j \right] \\ p_3^j - \rho_j B_j H_j \left[a_{o_j(j)3}^0 + \left({}^0\omega_1^j {}^0\omega_3^j - {}^0\dot{\omega}_2^j \right) x_1^j \right] \end{array} \right\}; \quad (23)$$

$$[V_j^1] = \begin{bmatrix} \frac{\rho_j B_j H_j}{12} (B_j^2 + H_j^2) & 0 & 0 \\ 0 & \frac{1049}{11340} \rho_j B_j H_j^3 & 0 \\ 0 & 0 & \frac{1049}{11340} \rho_j H_j B_j^3 \end{bmatrix}; \quad (24)$$

$$[V_j^2] = \begin{bmatrix} 0 & -\frac{\rho_j B_j H_j^3}{6} {}^0\omega_3^j & \frac{\rho_j H_j B_j^3}{6} {}^0\omega_2^j \\ \frac{\rho_j B_j H_j^3}{6} {}^0\omega_3^j & 0 & 0 \\ -\frac{\rho_j H_j B_j^3}{6} {}^0\omega_2^j & 0 & 0 \end{bmatrix}; \quad (25)$$

$$[V_j^3] = \begin{bmatrix} V_{j11}^3 & V_{j12}^3 & V_{j13}^3 \\ V_{j21}^3 & V_{j22}^3 & V_{j23}^3 \\ V_{j31}^3 & V_{j32}^3 & V_{j33}^3 \end{bmatrix}; \quad (26)$$

where:

$$V_{j11}^3 = -\frac{\rho_j B_j H_j^3}{12} ({}^0\omega_3^j)^2 - \frac{\rho_j H_j B_j^3}{12} ({}^0\omega_2^j)^2 - \frac{\rho_j B_j H_j}{12} (B_j^2 + H_j^2) ({}^0\omega_1^j)^2;$$

$$V_{j12}^3 = -\frac{\rho_j B_j H_j^3}{12} \left({}^0\omega_1^j {}^0\omega_2^j + {}^0\dot{\omega}_3^j \right);$$

$$V_{j13}^3 = -\frac{\rho_j H_j B_j^3}{12} \left({}^0\omega_1^j {}^0\omega_3^j - {}^0\dot{\omega}_2^j \right);$$

$$V_{j21}^3 = -\frac{\rho_j B_j H_j^3}{12} \left({}^0\omega_1^j {}^0\omega_2^j - {}^0\dot{\omega}_3^j \right);$$

$$V_{j22}^3 = -\frac{1049}{11340} \rho_j B_j H_j^3 \left[\left({}^0\omega_2^j \right)^2 + \left({}^0\omega_3^j \right)^2 \right] + \frac{247}{135} E_{1313} B_j H_j + \frac{4}{9} E_{1212} \frac{H_j^3}{B_j};$$

$$V_{j23}^3 = 0; V_{j31}^3 = -\frac{\rho_j H_j B_j^3}{12} \left({}^0\omega_1^j {}^0\omega_3^j + {}^0\dot{\omega}^j \right); V_{j32}^3 = 0;$$

$$V_{j33}^3 = -\frac{1049}{11340} \rho_j H_j B_j^3 \left[\left({}^0\omega_2^j \right)^2 + \left({}^0\omega_3^j \right)^2 \right] + \frac{247}{135} E_{1212} B_j H_j + \frac{4}{9} E_{1313} \frac{B_j^3}{H_j}; \quad (27)$$

$$[V_j^4] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{11}{9} E_{1331}^j B_j H_j \\ 0 & -\frac{11}{9} E_{1221}^j B_j H_j & 0 \end{bmatrix}; \quad (28)$$

$$[V_j^5] = \begin{bmatrix} -\frac{E_{1221} B_j H_j^3}{12} - \frac{E_{1331} H_j B_j^3}{12} & 0 & 0 \\ 0 & -\frac{1049}{11340} E_{1111} B_j H_j^3 & 0 \\ 0 & 0 & -\frac{1049}{11340} E_{1111} H_j B_j^3 \end{bmatrix}; \quad (29)$$

$$\{g_j\} = \left\{ \begin{array}{l} m_1^j + \frac{\rho_j B_j H_j^3}{12} \left({}^0\omega_2^j {}^0\omega_3^j - {}^0\dot{\omega}^j \right) - \frac{\rho_j H_j B_j^3}{12} \left({}^0\omega_2^j {}^0\omega_3^j + {}^0\dot{\omega}^j \right) \\ m_2^j - \frac{\rho_j B_j H_j^3}{12} \left({}^0\omega_1^j {}^0\omega_3^j + {}^0\dot{\omega}^j \right) \\ m_3^j + \frac{\rho_j H_j B_j^3}{12} \left({}^0\omega_1^j {}^0\omega_2^j - {}^0\dot{\omega}^j \right) \end{array} \right\}. \quad (30)$$

We have to consider the situation which the boundary points of bars are the places of the only existing couplings. In such a case the first derivative with respect to time of (27) of [13] leads us to:

$$\begin{aligned}
\left\{ \dot{A}^j \right\} &= \sum_{k=1}^j \left[{}^0 S^{k-1} \right]^t \left\{ r_{k-1}^{\ddot{\cdot}} \right\} + \left[{}^0 \dot{S}^{k-1} \right]^t \left\{ r_{k-1}^{\dot{\cdot}} \right\} + \left[{}^0 S^{k-1} \right]^t \left\{ r_k^{\dot{\cdot}} \right\} + \\
&+ \left[{}^0 \ddot{S}^{k-1} \right]^t \left\{ r_k^{k-1} \right\} - \left[{}^0 S^k \right]^t \left\{ r_{k-1}^{\ddot{\cdot}} \right\} - \left[{}^0 \dot{S}^k \right]^t \left\{ r_{k-1}^{\dot{\cdot}} \right\} - \left[{}^0 \dot{S}^k \right]^t \left\{ r_k^{\dot{\cdot}} \right\} - \\
&- \left[{}^0 \dot{S}^k \right]^t \left\{ r_k^{k-1} \right\}. \tag{31}
\end{aligned}$$

Remember that we are in the situation that the origin 0_j of T_j is placed in the boundary point L_{j-1} of the $j-1$ bar. In such a case we have:

$$\left\{ r_k^{k-1} \right\} = \{0\}_{3 \times 1}; \tag{32}$$

and $\left\{ r_k^{k-1} \right\}$ are algebraic vectors having constant value with respect to T_{k-1} , so, whatever derivative of them has to have zero value. So, (31) becomes:

$$\begin{aligned}
\left\{ \dot{A}^j \right\} &= \sum_{k=1}^j \left[{}^0 \ddot{S}^{k-1} \right] \left\{ r_{k-1}^{k-1} \right\} = \frac{d^2}{dt^2} \left(\sum_{k=1}^j \left[{}^0 S^{k-1} \right]^t \left\{ r_{k-1}^{k-1} \right\} \right) = \frac{d^2}{dt^2} \left\{ r_{0_j(0)}^0 \right\} = \\
&= \left\{ \ddot{r}_{0_j(0)}^0 \right\} = \left\{ a_{0_j(0)}^0 \right\}. \tag{33}
\end{aligned}$$

Obviously we have:

$$\left\{ a_{0_j(j)}^0 \right\} = \left[{}^0 S^j \right] \left\{ a_{0_j(0)}^0 \right\} = \left[{}^0 S^j \right] \left\{ \dot{A}^j \right\}. \tag{34}$$

4. CONCLUSIONS

An algorithm for establishing the mathematical model for the vibrations of each bar of the kinematic linkage. We propose such an algorithm which is consistent with all developments we have been working on. We shall attach to each

element of the kinematic linkage its own reference system related to its non-deformed status. We made consideration on how is best to choose such a system. We shall determine the generalized coordinates of the linkage, considering its bars as being rigid ones.

We shall determine every matrix $\left[{}^k S^j \right]$.

We shall determine every matrix: $\left[{}^0 \Omega^j \right] = \left[{}^0 \dot{S}^j \right] \cdot \left[{}^0 S^j \right]^t$.

We shall establish the position $A_{j,j-1}$, which means we shall determine the vectors \vec{r}_{j-1}^j and \vec{r}_j^{j-1} .

We shall make a full-complete kinematic and dynamic analysis for the linkage, considering bars that being still rigid. We shall calculate all matrix of the mathematical model according to relations (13) (30). We shall write the mathematical model in its classic form (7) and (8). We shall write all possible relations concerning the compatibility of displacements in all kinematic couplings. This kind of model can be improved in order to study more other sorts of bars like variable section composite bars and composite bars having some complex geometrical form. More, this sort of HSDT deformation hypothesis can well be put together with some sort of homogenization theory like this shown in [4, 5].

REFERENCES

1. Amirouché, F., *Computational Methods in Multibody Dynamics*, Prentice-Hall, 1992.
2. Attia, O., El-Zafrany, A., *A High Order Shear Element for Nonlinear Vibration Analysis of Composite Plates and Shells*, Pergamon, Int. J. Mech. Sci., **41**, 4–5, 461–486 (1999), Elsevier Science Ltd.
3. Barrau, J.J., Laroze, S., *Calcul des structures en matériaux composites*, **4**, Ecole Nationale de l'aéronautique et de l'espace, Toulouse, 1987.
4. Bolcu, D., Stănescu, G., Ursache, M., *Theoretical and experimental study on determination of the elastic properties of the composite materials*, Romanian Reports in Physics, **56**, 1, 3–12 (2004).
5. Bolcu, D., Stănescu, G., Ursache, M., *Determination of elasticity modulus for the composite materials based on resin-textile*, Analele Universității din Craiova, Physics AUC, **14**, 47–54 (2004).
6. Gay, D., *Matériaux composites*, Edition Hermes, Paris, 1989.
7. Gordaninejad, F., Ghazavi, A., Chalhoub, N.G., *Nonlinear Dynamic Modelling of a Revolute – Prismatic Flexible Composite – Material Robot Arm*, Jour. Vibr. Acous., **113**, 461–468 (1991).
8. Librescu, L., *Statica și dinamica structurilor elastice, anizotrope și eterogene*, Edit. Academiei Române, 1969.
9. Librescu, L., Thangjithan, S., *Parametric instability laminated composite shear-deformable flat panels subject to in-plane loads*, Ind. J. Non-linear Mechanics, **25**, 263–273 (1990).

10. Reddy, J.N., *Energy and Variational Methods in Applied Mechanics*, John Wiley and Sons, New York, 1984.
11. Răzescu, S., Bolcu, D., Stănescu, G., *A new deformation hypothesis used to model the elastic behavior of the prismatic composite bars with elastic symmetry*, *Mashinintelekt*, 12 (2004).
12. Răzescu, S., Rinderu, P., Bolcu, D., *Ecuatii de mișcare pentru un element cinematic de tip bară compozită*, Conferința Națională de Robotică, Craiova, 2002.
13. Răzescu, S., Ursache, M., *The calculus of the kinetic energy for a kinematic linkage made of composite bars using an improved HSDT-deformation hypothesis*, *Analele Universității din Craiova, seria Fizică, Physics AUC*, 17 (Part II), 124–131 (2007).
14. Răzescu S., Ursache M., *The calculus of the deformation mechanical work for a kinematic linkage made of composite bars using an improved HSDT-deformation hypothesis*, *Analele Universității din Craiova, seria Fizică, Physics AUC*, 17 (Part II), 132–139 (2007).
15. Stăicu, S., *Aplicații ale calculului matriceal în mecanica solidelor*, Edit. Academiei Române, 1986.