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# POISSON-TYPE FIELD EQUATIONS OF EINSTEINIAN GRAVITOSTATICS

NICHOLAS IONESCU-PALLAS

*former Senior Scientific Researcher of the Institute for Atomic Physics*
*Măgurele Campus, Bucharest, Romania*

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*Abstract*. Poisson-type field equations, coming from coordinate conditions $\nu + \sigma = 0$, is assumed, and a selfconsistent Gravitostatics is substantiated in the framework of General Theory of Relativity. This implied the adoption of a point-like black hole concept, associated with an invariant whose physical dimensions are those of a surface. Full compliance with black-hole thermodynamics is ensured. In the first relativistic approximation, the exterior metric is given as a function of the potential Φ, as expected.

*Key words:* Einsteinian gravitostatics, point-like black hole.

The suitable framework for successfully entering upon gravitational problems is the General Relativity Theory put forward by Albert Einstein in 1916

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{8\pi G}{c^4} T_{\alpha\beta} . \tag{1}$$

We look for the solution of these equations in the case of a spherical body, under the following circumstances: 1) polar coordinates ($r$, θ, φ) are used; 2) mass distribution within the source sphere of radius $R_s$ has spherical symmetry; 3) perfect fluid scheme is adopted for defining the structure of the matter tensor $T_{\alpha\beta}$; 4) mass distribution $\rho(r)$ is arbitrarily given and the pressure within the source is determined out of the mechanical equilibrium condition; 5) in the case of a point-like source, located in the origin of the inertia frame, the entire three-dimensional space of positions is accessible to observation, apart for the point $r = 0$.

Taking the origin of the reference frame to coincide with the center of the source sphere, the solution of the problem is the 4-dimensional metric, which, owing to the spherical symmetry, has the form

$$\begin{aligned}(\mathrm{d}S)^2 = A(r)(c\mathrm{d}t)^2 - \Big\{ B(r)(\mathrm{d}r)^2 + C(r)r^2 \Big[ (\mathrm{d}\theta)^2 + \sin^2\theta(\mathrm{d}\varphi)^2 \Big] \Big\} \\ + D(r)(c\mathrm{d}t)\mathrm{d}r \end{aligned} \tag{2}$$

The physical meaning of the quantity $(\mathrm{d}S)^2$, often called "the metric of the Universe", is coming from its connection to the Lagrange function $L$, describing the motion of a test point-like body in the gravitational static field yielded by the source sphere, through the agency of the formula

$$L = -m_0 c^2 \frac{\mathrm{d}S}{c\mathrm{d}t}. \tag{3}$$

In the limiting case $r \to \infty$, we have to recover the known formula of a free body Lagrangean

$$L = -m_0 c^2 (1 - \frac{\vec{v}^2}{c^2})^{1/2}, \tag{4}$$

whence the conditions

$$\lim_{r\to\infty} A(r) = 1, \quad \lim_{r\to\infty} B(r) = 1, \quad \lim_{r\to\infty} C(r) = 1, \quad \lim_{r\to\infty} D(r) = 0. \tag{5}$$

We may determine the unknown function $D(r)$ by resorting to formula (3) for asking the holding of the correspondence principle between the Classical Mechanics and the Relativistic one, under the form

$$\lim_{\vec{v}\to 0} \vec{p} = \lim_{\vec{v}\to 0} \frac{\partial L}{\partial \vec{v}} = 0, \tag{6}$$

whence

$$D(r) = 0. \tag{7}$$

For the remained unknown functions, it is convenient to make the chances

$$A(r) = \mathrm{e}^{\nu(r)}, \quad B(r) = \mathrm{e}^{\lambda(r)}, \quad C(r) = \mathrm{e}^{\sigma(r)}, \tag{8}$$

so that the metric $(\mathrm{d}S)^2$ acquires the form

$$(\mathrm{d}S)^2 = \mathrm{e}^{\nu}(c\mathrm{d}t)^2 - \left\{ \mathrm{e}^{\lambda}(\mathrm{d}r)^2 + \mathrm{e}^{\sigma} r^2 \left[ (\mathrm{d}\theta)^2 + \sin^2\theta (\mathrm{d}\varphi)^2 \right] \right\}. \tag{9}$$

In the perfect fluid scheme, the expression of the matter tensor is given by the formula

$$T_{\alpha\beta} = (c^2 + H)\rho U_\alpha U_\beta - p g_{\alpha\beta}, \tag{10}$$

where

$$H = \int_0^{p(\rho)} \frac{\mathrm{d}p}{\rho(p)} \tag{11}$$

is the Helmholtz potential of the source fluid.

The non-vanishing components of the tensors $E_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$ and $T_{\alpha\beta}$ are ($E_{00}$, $E_{11}$, $E_{22}$, $E_{33}$), respectively ($T_{00}$, $T_{11}$, $T_{22}$, $T_{33}$). But, there are the relations

$$E_{33} = E_{22}\sin^2\theta\ ,\ \ T_{33} = T_{22}\sin^2\theta, \tag{12}$$

so that the system of equations (1) reduces to three distinct equations

$$E_{00} + \frac{8\pi G}{c^4}T_{00} = 0,\quad E_{11} + \frac{8\pi G}{c^4}T_{11} = 0,\quad E_{22} + \frac{8\pi G}{c^4}T_{22} = 0. \tag{13}$$

Apparently, we have at our disposal three equations (13) for determining three unknown functions of the metric (9). However, this is not the case, because of the existence of conservativity constraints of the tensors ($E_{\alpha\beta}$, $T_{\alpha\beta}$), namely

$$\nabla_\eta (g^{\eta\alpha} g^{\zeta\beta} E_{\alpha\beta}) \equiv 0,\quad \nabla_\eta (g^{\eta\alpha} g^{\zeta\beta} T_{\alpha\beta}) = 0. \tag{14}$$

In detailed form, these conditions lead to formulas

$$\begin{aligned}
&(\mathrm{e}^{-2\lambda} E_{11})' + \frac{1}{2}\nu' \mathrm{e}^{-(\nu+\lambda)} E_{00} + \mathrm{e}^{-2\lambda} E_{11} (\sigma' + \frac{2}{r} + \lambda' + \frac{1}{2}\nu') - \\
&- \frac{1}{r^2} \mathrm{e}^{-(\sigma+\lambda)} E_{22} (\sigma' + \frac{2}{r}) = 0, \\
&(\mathrm{e}^{-2\lambda} T_{11})' + \frac{1}{2}\nu' \mathrm{e}^{-(\nu+\lambda)} T_{00} + \mathrm{e}^{-2\lambda} T_{11} (\sigma' + \frac{2}{r} + \lambda' + \frac{1}{2}\nu') - \\
&- \frac{1}{r^2} \mathrm{e}^{-(\sigma+\lambda)} T_{22} (\sigma' + \frac{2}{r}) = 0.
\end{aligned} \tag{15}$$

Taking into account Eqs. (15), we conclude that only two equations of the system (13) are independent – the third one being a mathematical consequence of the other two. As the Einstein's Theory *turns out to be incomplete*, for solving the gravitostatic problem we search for, we need to introduce a supplementary condition, which is beyond the theory. We assume this additional condition to be just the Poisson-type form of the field equations.

In 1916, K. Schwarzschild arbitrarily assumed $\sigma = 0$ and determined the remained functions of the metric as

$$\mathrm{e}^{\nu} = \mathrm{e}^{-\lambda} = 1 - 2\frac{\mu}{r},\quad \mu = \frac{GM_0}{c^2},\quad r \geq R_s,\quad \sigma = 0. \tag{16}$$

The most strange consequence of the Schwarzschild metric is the existence of a singular sphere, of radius $r_s = 2\mu$, surrounding a point-like source (the so call "black hole").

The expressions of the non-vanishing components of the tensors $E_{\alpha\beta}$ and $T_{\alpha\beta}$ are given in the sequel

$$
\begin{aligned}
&\mathrm{e}^{\lambda-\nu}E_{00} = \Delta\sigma + \frac{3}{4}\sigma'^2 + \frac{1}{r}(\sigma'-\lambda') - \frac{1}{2}\lambda'\sigma' + \frac{1}{r^2}(1-\mathrm{e}^{\lambda-\sigma}) = -\frac{8\pi G}{c^4}T_{00}\mathrm{e}^{\lambda-\nu}\\
&-E_{11} = +\frac{1}{4}\sigma'^2 + \frac{1}{r}(\sigma'+\nu') + \frac{1}{2}\nu'\sigma' + \frac{1}{r^2}(1-\mathrm{e}^{\lambda-\sigma}) = +\frac{8\pi G}{c^4}T_{11}\\
&\frac{1}{r^2}\mathrm{e}^{\lambda-\sigma}E_{22} = -\frac{1}{2}\left(\Delta\sigma + \frac{1}{2}\sigma'^2\right) - \frac{1}{2}\left(\Delta\nu + \frac{1}{2}\nu'^2\right) + \frac{1}{2}\cdot\frac{1}{r}(\nu'+\lambda')\\
&+\frac{1}{4}\sigma'(\lambda'-\nu') + \frac{1}{4}\nu'\lambda' = -\frac{8\pi G}{c^4}\frac{1}{r^2}\mathrm{e}^{\lambda-\sigma}T_{22}\\
&\mathrm{e}^{-\nu}T_{00} = c^2\rho + (H\rho - p), \qquad \mathrm{e}^{-\lambda}T_{11} = p, \qquad \frac{1}{r^2}\mathrm{e}^{-\sigma}T_{22} = p.
\end{aligned}
\tag{17}
$$

It is ascertained that the only quantities containing second order derivatives are included in the terms $\Delta\sigma$ and $\Delta\nu$. If we ask the condition

$$
\sigma = -\nu, \tag{18}
$$

then a Laplace-type term will remain only in the first equation (17) and the system (13) will be readily integrated. The equations (17) become

$$
\begin{aligned}
&\mathrm{e}^{\lambda-\nu}E_{00} = -\Delta\nu + \frac{3}{4}\nu'^2 - \frac{1}{r}(\nu'+\lambda') + \frac{1}{2}\nu'\lambda' + \frac{1}{r^2}(1-\mathrm{e}^{\lambda+\nu}) = -\frac{8\pi G}{c^4}T_{00}\mathrm{e}^{\lambda-\nu}\\
&-E_{11} = -\frac{1}{4}\nu'^2 + \frac{1}{r^2}(1-\mathrm{e}^{\lambda+\nu}) = +\frac{8\pi G}{c^4}T_{11},
\end{aligned}
\tag{19a}
$$

$$
\begin{aligned}
&\frac{1}{r^2}\mathrm{e}^{\lambda+\nu}E_{22} = -\frac{1}{4}\nu'^2 + \frac{1}{2}\frac{1}{r}(\nu'+\lambda') = -\frac{8\pi G}{c^4}\frac{1}{r^2}\mathrm{e}^{\lambda+\nu}T_{22},\\
&\mathrm{e}^{-\nu}T_{00} = c^2\rho + (H\rho - p), \qquad \mathrm{e}^{-\lambda}T_{11} = p, \qquad \frac{1}{r^2}\mathrm{e}^{\nu}T_{22} = p.
\end{aligned}
\tag{19b}
$$

Making up the combination $E_{11} + \frac{1}{r^2}\mathrm{e}^{\lambda+\nu}E_{22}$ in (19) and taking into account the equations (13) we come to the integrable equation

$$
\frac{16\pi G}{c^4}p\mathrm{e}^{\lambda} + \frac{1}{2r}(\nu'+\lambda') + \frac{1}{r^2}(\mathrm{e}^{\lambda+\nu} - 1) = 0\,, \tag{20}
$$

which may be integrated to give

$$
\mathrm{e}^{-(\nu+\lambda)} = 1 + \frac{1}{r^2}F(r), \qquad F(r) = \frac{32\pi G}{c^4}\int_0^r r^3 p\mathrm{e}^{-\nu}\mathrm{d}r\,. \tag{21}
$$

Further on, we may eliminate the terms $\frac{1}{r}(\nu'+\lambda')$ and $\frac{1}{r^2}(1-\mathrm{e}^{\lambda+\nu})$ from the first equation (19) by resorting to the second and the third equation (19). Thereafter, we replace the components of $E_{\alpha\beta}$, in the resulting equation by the components of $T_{\alpha\beta}$, using equations (13). So, one obtains

$$\Delta\nu = \frac{8\pi G}{c^4}\mathrm{e}^{\lambda}\left[\rho(c^2+H)+2p\right]+\frac{1}{2}\nu'(\nu'+\lambda') . \quad (22)$$

But $\Delta\nu$ may be expressed as

$$\Delta\nu \equiv \frac{2}{\mathrm{ch}\,\frac{\nu}{2}}\Delta\mathrm{sh}\,\frac{\nu}{2}-\frac{1}{2}\nu'^2\ \mathrm{th}\,\frac{\nu}{2} . \quad (23)$$

So that, eliminating $\Delta\nu$ between (22) and (23) we come to the Poisson-type equation

$$\begin{gathered}\Delta\mathrm{sh}\,\frac{\nu}{2}=\frac{4\pi G}{c^4}\rho_E \\ \rho_E \equiv (\mathrm{e}^{\lambda}\mathrm{ch}\,\frac{\nu}{2})\left[\rho\ \ c^2+(\rho H+2p)\right]+\frac{c^4}{16\pi G}\nu'(\nu'\mathrm{e}^{\frac{1}{2}\nu}+\lambda'\mathrm{ch}\,\frac{\nu}{2}).\end{gathered} \quad (24)$$

We have now to demonstrate that the above-differential equation is really a Poisson-type equation, this implying $\rho_E = 0$ outside the mass distribution. For checking this statement, we need to build up the field solution for $r \geq R_S$.

For $r > R_S$ the second equation (19) and the equation (21) become

$$\begin{gathered}\frac{1}{r^2}(1-\mathrm{e}^{\lambda+\nu})-\frac{1}{4}\nu'^2=0, \qquad \mathrm{e}^{-(\nu+\lambda)}=1+\left(\frac{k}{r}\right)^2 , \\ k^2 \equiv \int_0^{R_S}\frac{32\pi G}{c^4}r^3 p\mathrm{e}^{-\nu}\mathrm{d}r.\end{gathered} \quad (25)$$

Eliminating $(\nu+\lambda)$ between the two equations (25) and integrating the resulting equation, one obtains

$$\nu = 2\ln\left(\sqrt{1+f^2}-f\right), \qquad f \equiv \frac{k}{r} . \quad (26)$$

From (26) we further obtain

$$f = -\mathrm{sh}\,\frac{\nu}{2}, \qquad \Delta\mathrm{sh}\,\frac{\nu}{2}=4\pi k\delta(\vec{r}) . \quad (27)$$

This explaining why we used the identity (23). The metric functions for the exterior space are now

$$\mathrm{e}^{\nu}=\mathrm{e}^{-\sigma}=\left(\sqrt{1+f^2}-f\right)^2 , \qquad \mathrm{e}^{-\lambda}=\mathrm{e}^{\nu}\left(1+f^2\right). \quad (28)$$

Thus, the *exterior metric has only a point-like singularity*. This is not strange at all, because the classical Lagrange function for a point-like source body has such a singularity as well. Using now (28), we prove that, for $r > R_S$,

$$\nu'\mathrm{e}^{\frac{1}{2}\nu}+\lambda'\mathrm{ch}\,\frac{\nu}{2}=0 \quad (29)$$

and, accordingly, $\rho_E = 0$ for $r > R_S$. Finally, we define a potential through formula

$$\Phi = -\operatorname{sh}\frac{\nu}{2}. \tag{30}$$

Outside the mass distribution Φ goes into f provided that

$$k = \frac{GE}{c^4}. \tag{31}$$

The metric, defined all over the position space, has the expression

$$\begin{aligned}(\mathrm{d}S)^2 &= \left(\sqrt{1+\Phi^2}-\Phi\right)^2 (c\mathrm{d}t)^2 - \left(\sqrt{1+\Phi^2}+\Phi\right)^2 \times \\ &\times \left\{\frac{r^2}{r^2+F}(\mathrm{d}r)^2 + r^2\left[(\mathrm{d}\vartheta)^2 + \sin^2\vartheta(\mathrm{d}\varphi)^2\right]\right\},\end{aligned} \tag{32}$$

where $F(r)$ is defined in (21) and Φ fulfils the Poisson-type equation

$$\Delta\Phi = -\frac{4\pi G}{c^4}\rho_E. \tag{33}$$

Two problems remained to be solved – first to prove the equality (31) and second to demonstrate that the quantity E in that equality, defined as

$$E = \int_0^{R_s} 4\pi r^2 \rho_E \mathrm{d}r, \tag{34}$$

Is just the quantity labeled as "energy" in the conventional Physics based on a flat chronotopic Universe. For these purposes, we return to the second equation (15) and insert there the expressions of the components $T_{\alpha\beta}$ taken out from (17). So, we come to the equation

$$p' + \frac{1}{2}\nu'\rho(c^2 + H) = 0. \tag{35}$$

At the same time, from (11) we obtain as well

$$p' - \rho H' = 0. \tag{36}$$

Eliminating p' between (35) and (36) and integrating the resulting equation under the limiting condition $H(R_S) = 0$, we come to the formula

$$H(r) = c^2\left\{\mathrm{e}^{\frac{1}{2}[\nu(R_s)-\nu(r)]} - 1\right\}. \tag{37}$$

Thereafter, we introduce $H(r)$ in explicit form, taken out from (37), in one of the two equations (35), (36) and integrate again for rendering $p$, in terms of ν. Whence

$$p(r) = \frac{1}{2}c^2\int_r^{R_s} \mathrm{e}^{\frac{1}{2}[\nu(R_s)-\nu(r)]}\rho(r)\nu'(r)\mathrm{d}r. \tag{38}$$

(The limiting condition $p(R_S)$ was imposed). Further on, we assume the fluid of the source as made up of $N$ identical particles with rest mass $m_0$. Moreover, we admit that the elementary number d$N$ of such particles is invariant under the going over from the curved to the flat (Minkowski) Universe

$$\frac{1}{m_0}\rho\sqrt{-g}\,\frac{c\mathrm{d}t}{\mathrm{d}S}\left(\mathrm{d}^3x\right)=\frac{1}{m_0}\rho_M\sqrt{-g_M}\,\frac{c\mathrm{d}t}{\mathrm{d}S_M}\left(\mathrm{d}^3x\right)=\mathrm{d}N. \tag{39}$$

(The quantities defined in the Minkowski Universe were marked by a subscript M). From (39) we infer the formula

$$\rho=\mathrm{e}^{-(\sigma+\frac{1}{2}\lambda)}\rho_M. \tag{40}$$

Now, we may express the pressure only in terms of observable quantities

$$p=-c^2\left(\sqrt{1+\frac{\mu^2}{R_s^2}}-\frac{\mu}{R_s}\right)\int_r^{R_s}\rho_M\Phi'\left(\frac{1+\dfrac{F}{r^2}}{1+\Phi^2}\right)^{\frac{1}{2}}\left(\sqrt{1+\Phi^2}-\Phi\right)^2\mathrm{d}r. \tag{41}$$

Actually, equation (41) is an intricate integral equation defining pressure in terms of potential. However, this equation is suitable for useful approximations.

In a non relativistic approximation one obtains

$$p=-c^2\int_r^{R_s}\rho_M\Phi'\mathrm{d}r+O\left(\frac{1}{c^2}\right). \tag{42}$$

In the foregoing demonstrations, we redefine the potential for a closer analogy with the Newtonian Theory of Gravitation, namely

$$\begin{aligned}&U(r)=c^2\Phi\\&\Delta U(r)=-4\pi G\rho_m,\quad \rho_m=\frac{1}{c^2}\rho_E.\end{aligned} \tag{43}$$

Now, we may write

$$\begin{aligned}&p=-\int_r^{R_s}\rho_m U'\mathrm{d}r+O\left(\frac{1}{c^2}\right),\\&U(r)=\frac{G}{r}\int_0^r 4\pi\rho_m r^2\mathrm{d}r+G\int_r^{R_s}4\pi\rho_m r\mathrm{d}r.\end{aligned} \tag{44}$$

The following identity may easily be proved

$$\int_0^{R_s}\left(\frac{1}{2}\rho_m U-3p\right)4\pi\ \ r^2\mathrm{d}r=0+O\left(\frac{1}{c^2}\right). \tag{45}$$

At the same time, some useful approximations are to be noticed

$$\nu = -2\frac{U}{c^2} + O\left(\frac{1}{c^4}\right), \quad \lambda = +2\frac{U}{c^2} + O\left(\frac{1}{c^4}\right),$$
$$H = U - \frac{GM_s}{R_s} + O\left(\frac{1}{c^2}\right). \tag{46}$$

The energy density in (24) may be now approximated as

$$\rho_E = \rho_M c^2 \left(1 - \frac{GM_s}{c^2 R_s}\right) + 2p + O\left(\frac{1}{c^2}\right). \tag{47}$$

The expression we expect to obtain, according to the Fluidodynamics devised in a flat Universe is

$$\rho_E^{(M)} = \rho_M c^2 \left(1 - \frac{GM_s}{c^2 R_s}\right) + \left(\frac{1}{2}\rho_m U - p\right) + O\left(\frac{1}{c^2}\right). \tag{48}$$

Subtracting, part by part, (47) from (48) and accounting for (45) we conclude that

$$\int_0^{R_{ss}} \rho_E \, 4\pi r^2 \mathrm{d}r = \int_0^{R_s} \rho_E^{(M)} \, 4\pi r^2 \mathrm{d}r + O\left(\frac{1}{c^2}\right). \tag{49}$$

Further on, from (44) and (21) one obtains

$$F(r) = \left(\frac{GE}{c^4}\right)^2 \left(\frac{r}{R_s}\right)^4 \left[1 + \left(\frac{2c^2 R_s^2}{GE}\right)^2 \int_r^{R_s} U'^2 \frac{\mathrm{d}r}{r}\right] + O\left(\frac{1}{c^6}\right). \tag{50}$$

Another equivalent approximation is

$$F(r) = \frac{G^2}{c^4}\left\{\left(\int_0^r 4\pi r^2 \rho_m \mathrm{d}r\right) + 2r^4 \int_r^{R_s} 4\pi\rho_m \frac{\mathrm{d}r}{r^2} \int_0^r 4\pi \;\; r^2 \rho_m \mathrm{d}r\right\} + O\left(\frac{1}{c^6}\right). \tag{51}$$

Out of (50) or (51) the equality (31) is approximately checked. For obtaining an accurate result we proceed as follows. We add together the second and the third equation in (19) and take into account that

$$\frac{1}{r^2}\mathrm{e}^{\lambda+\nu} E_{22} - E_{11} = -\frac{8\pi G}{c^4}\left(\frac{1}{r^2}\mathrm{e}^{\lambda+\nu} T_{22} - T_{11}\right) = 0. \tag{52}$$

Thus, one obtains the equation

$$\frac{1}{r^2}\left(1 - \mathrm{e}^{\lambda+\nu}\right) + \frac{1}{2r}(\lambda+\nu)' - \frac{1}{2}\nu'^2 = 0. \tag{53}$$

Replacing in this equation the quantity $(\lambda+\nu)$ by its expression in terms of $F$, taken out from (21) we come to the equation

$$F(4-\nu'^2 r^2)-(rF'+r^4\nu'^2)=0. \tag{54}$$

But, taking the derivative in the integral representation of $F$ in (21), we obtain

$$rF'=\frac{32\pi G}{c^4}r^4 p\mathrm{e}^{-\nu}. \tag{55}$$

Finally, eliminating $rF'$ between (54) and (55), we come to the result

$$F=r^4\cdot\frac{\nu'^2+\dfrac{32\pi G}{c^4}p\mathrm{e}^{-\nu}}{4-\nu'^2\cdot r^2}. \tag{56}$$

We may still express ν and ν' in terms of the potential Φ. So, we have

$$F(r)=(r^2\Phi'^2)^2\,\frac{1+\dfrac{8\pi G}{c^4}p\dfrac{1+\Phi^2}{\Phi'^2}(\sqrt{1+\Phi^2}+\Phi)^2}{1+\Phi^2-(r\Phi')^2}. \tag{57}$$

Now, we are able to calculate the limit

$$\begin{aligned}&\lim_{r\to R_s}F(r)=\lim_{r\to R_s}\left(r^2\Phi'\right)^2=\left(\frac{G}{c^4}\right)^2\left(\lim_{r\to R_s}\int_0^r 4\pi\rho_E r^2\mathrm{d}r^2\right)=\\&=\left(\frac{GE}{c^4}\right)^2=\frac{32\pi G}{c^4}\int_0^{R_s}r^3 p\mathrm{e}^{-\nu}\mathrm{d}r,\quad Q.E.D.\end{aligned} \tag{58}$$

In the first relativistic approximation, the function $F$ does not enter the metric, and we obtain a functional approximation in the framework of Einstein's General Relativity Theory

$$\left(\mathrm{d}S\right)^2\approx\left(1-2\Phi+2\Phi^2\right)\left(c\mathrm{d}t\right)^2-\left(1+2\Phi\right)\left[\left(\mathrm{d}x\right)^2+\left(\mathrm{d}y\right)^2+\left(\mathrm{d}z\right)^2\right]. \tag{59}$$

No contradiction we came across in devising the solution without "black hole". As we see, the solution is approximately isotropic, existing in this respect a certain resemblance with the Fock's solution. We point out that we refer to the conventional picture of a black hole, as it is delivered by the Schwarzschild metric. In our approach, based on the accessible space conjecture, the black hole concept is retained only as point-like singularity. As the surface of the point-like black hole is just $S = 16\,\pi\mu^2$ (as in the Schwarzschild case) no contradiction occurs with the Hawking's thermodynamics of black holes.

In the first order relativistic approximation, the metric (59) is more convenient for astrophysical calculations, as compared with the Schwarzschild metric in the same approximation

$$(\mathrm{d}S)^2 \approx (1-2\Phi)(c\mathrm{d}t)^2 - \left\{(1-2r\Phi')\frac{(\vec{r}\cdot\mathrm{d}\vec{r})^2}{r^2} + \frac{(\vec{r}\times\mathrm{d}\vec{r})^2}{r^2}\right\} \qquad (60)$$

because the metric (59) depends only on Φ.

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